جلسه یازدهم

مکانیک تحلیلی

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اسفند ۹۸

مکانیک لاگرانژی

یادآوری

$$x = x(\mathbf{q}),$$
$$y = y(\mathbf{q}),$$
$$z = z(\mathbf{q})$$

$$Q_k = F_x \frac{\partial x}{\partial q_k} + F_y \frac{\partial y}{\partial q_k} + F_z \frac{\partial z}{\partial q_k}$$

$$\vec{F} = -\vec{\nabla} V, \qquad Q_k = -\frac{\partial V}{\partial q_k}$$

$$\dot{x} = \dot{x}(\mathbf{q}, \dot{\mathbf{q}}) = \sum_k \frac{\partial x}{\partial q_k} \dot{q}_k, \qquad \dot{z} = \dot{z}(\mathbf{q}, \dot{\mathbf{q}}) = \sum_k \frac{\partial z}{\partial q_k} \dot{q}_k$$

$$\dot{y} = \dot{y}(\mathbf{q}, \dot{\mathbf{q}}) = \sum_k \frac{\partial y}{\partial q_k} \dot{q}_k, \qquad T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

مکانیک لاگرانژی

بررسی درستی روابط،

یادآوری

$$\frac{\partial \dot{x}}{\partial \dot{q}_k} = \frac{\partial x}{\partial q_k}, \qquad \frac{\partial \dot{y}}{\partial \dot{q}_k} = \frac{\partial y}{\partial q_k}, \qquad \frac{\partial \dot{z}}{\partial \dot{q}_k} = \frac{\partial z}{\partial q_k}$$

$$\dot{x} = \dot{x}(\mathbf{q}, \dot{\mathbf{q}}) = \sum_k \frac{\partial x}{\partial q_k} \dot{q}_k,$$

$$\frac{\partial}{\partial \dot{q}_l} \dot{x} = \frac{\partial}{\partial \dot{q}_l} \sum_k \frac{\partial x}{\partial q_k} \dot{q}_k \Rightarrow \frac{\partial \dot{x}}{\partial \dot{q}_l} = \sum_k \left(\frac{\partial^2 x}{\partial \dot{q}_l \partial q_k} \dot{q}_k + \frac{\partial x}{\partial q_k} \frac{\partial \dot{q}_k}{\partial \dot{q}_l} \right)$$

$$x = x(\mathbf{q}): \quad \frac{\partial^2 x}{\partial \dot{q}_l \partial q_k} = \frac{\partial^2 x}{\partial q_k \partial \dot{q}_l} = \frac{\partial}{\partial q_k} \left(\frac{\partial x}{\partial \dot{q}_l} \right) = 0, \qquad \frac{\partial \dot{q}_k}{\partial \dot{q}_l} = \delta_{k,l}$$

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$$\frac{\partial \dot{x}}{\partial \dot{q}_l} = \sum_k \left(\frac{\partial^2 x}{\partial q_k \partial \dot{q}_l} \dot{q}_k + \frac{\partial x}{\partial q_k} \frac{\partial \dot{q}_k}{\partial \dot{q}_l} \right)$$

$$x = x(\mathbf{q}) : \frac{\partial^2 x}{\partial q_k \partial \dot{q}_l} = 0, \qquad \frac{\partial \dot{q}_k}{\partial \dot{q}_l} = \delta_{k,l}$$

$$\frac{\partial \dot{x}}{\partial \dot{q}_l} = \sum_k \frac{\partial x}{\partial q_k} \delta_{k,l}$$

$$\frac{\partial \dot{x}}{\partial \dot{q}_l} = \frac{\partial x}{\partial q_l}$$

مکانیک لاگرانژی

دستگاه ذرات

$$
\begin{array}{c|c|c|c}
 & F_1^x = m_1\ddot{x}_1 & F_1^x = m_1\ddot{x}_1 & F_1^x = m_1\ddot{x}_1 \\
m_1 : (x_1, y_1, z_1) & F_2^x = m_2\ddot{x}_2 & F_2^x = m_2\ddot{x}_2 & F_2^x = m_2\ddot{x}_2 \\
\vdots & \vdots & \vdots & \vdots \\
m_i : (x_i, y_i, z_i) & F_i^x = m_i\ddot{x}_i & F_i^x = m_i\ddot{x}_i & F_i^x = m_i\ddot{x}_i \\
\vdots & \vdots & \vdots & \vdots \\
m_N : (x_N, y_N, z_N) & F_N^x = m_N\ddot{x}_N & F_N^x = m_N\ddot{x}_N & F_N^x = m_N\ddot{x}_N
\end{array}
$$

درجه آزادی : $3N$

$$T = \sum_i^N \frac{1}{2} m_i (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)$$

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دستگاه ذرات

$$m_1 : (x_1, y_1, z_1)$$
$$m_2 : (x_2, y_2, z_2)$$
$$\vdots$$
$$m_i : (x_i, y_i, z_i)$$
$$\vdots$$
$$m_N : (x_N, y_N, z_N)$$

$3N$: درجه آزادی

$$\mathbf{q} = (q_1, q_2, \cdots, q_k, \cdots, q_n)$$

n : تعداد مختصه‌های تعمیم یافته

$$n \leq 3N$$

قرارداد:

i : شمارنده ذرات $\quad 1 \leq i \leq N$

k : شمارنده مختصه‌های تعمیم یافته $\quad 1 \leq k \leq n$

مکانیک لاگرانژی

ذره‌ی i ام

$$\frac{\partial \dot{x}_i}{\partial \dot{q}_k} = \frac{\partial x_i}{\partial q_k} \quad \xrightarrow{\times \dot{x}_i} \quad \dot{x}_i \frac{\partial \dot{x}_i}{\partial \dot{q}_k} = \dot{x}_i \frac{\partial x_i}{\partial q_k} \quad \xrightarrow{\times m_i} \quad m_i \dot{x}_i \frac{\partial \dot{x}_i}{\partial \dot{q}_k} = m_i \dot{x}_i \frac{\partial x_i}{\partial q_k}$$

$$\frac{\partial \dot{y}_i}{\partial \dot{q}_k} = \frac{\partial y_i}{\partial q_k} \quad \xrightarrow{\times \dot{y}_i} \quad \dot{y}_i \frac{\partial \dot{y}_i}{\partial \dot{q}_k} = \dot{y}_i \frac{\partial y_i}{\partial q_k} \quad \xrightarrow{\times m_i} \quad m_i \dot{y}_i \frac{\partial \dot{y}_i}{\partial \dot{q}_k} = m_i \dot{y}_i \frac{\partial y_i}{\partial q_k}$$

$$\frac{\partial \dot{z}_i}{\partial \dot{q}_k} = \frac{\partial z_i}{\partial q_k} \quad \xrightarrow{\times \dot{z}_i} \quad \dot{z}_i \frac{\partial \dot{z}_i}{\partial \dot{q}_k} = \dot{z}_i \frac{\partial z_i}{\partial q_k} \quad \xrightarrow{\times m_i} \quad m_i \dot{z}_i \frac{\partial \dot{z}_i}{\partial \dot{q}_k} = m_i \dot{z}_i \frac{\partial z_i}{\partial q_k}$$

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ذره‌ی i ام

$$m_i\dot{x}_i\frac{\partial\dot{x}_i}{\partial\dot{q}_k}=m_i\dot{x}_i\frac{\partial x_i}{\partial q_k}$$

$$m_i\dot{y}_i\frac{\partial\dot{y}_i}{\partial\dot{q}_k}=m_i\dot{y}_i\frac{\partial y_i}{\partial q_k}$$

$$m_i\dot{z}_i\frac{\partial\dot{z}_i}{\partial\dot{q}_k}=m_i\dot{z}_i\frac{\partial z_i}{\partial q_k}$$

ساده سازی

$$\frac{\partial}{\partial\dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)=m_i\dot{x}_i\frac{\partial x_i}{\partial q_k}$$

$$\frac{\partial}{\partial\dot{q}_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)=m_i\dot{y}_i\frac{\partial y_i}{\partial q_k}$$

$$\frac{\partial}{\partial\dot{q}_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)=m_i\dot{z}_i\frac{\partial z_i}{\partial q_k}$$

مکانیک لاگرانژی

ذره‌ی i ام

$\frac{\mathrm{d}}{\mathrm{d}t}$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{x}_i\frac{\partial x_i}{\partial q_k}\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{y}_i\frac{\partial y_i}{\partial q_k}\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)\right] = \frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{z}_i\frac{\partial z_i}{\partial q_k}\right]$$

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مکانیک لاگرانژی

ذره‌ی i ام

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{x}_i\frac{\partial x_i}{\partial q_k}\right]=m_i\ddot{x}_i\frac{\partial x_i}{\partial q_k}+m_i\dot{x}_i\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial x_i}{\partial q_k}\right]=F_i^x\frac{\partial x_i}{\partial q_k}+m_i\dot{x}_i\frac{\partial \dot{x}_i}{\partial q_k}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{x}_i\frac{\partial x_i}{\partial q_k}\right]=F_i^x\frac{\partial x_i}{\partial q_k}+m_i\dot{x}_i\frac{\partial \dot{x}_i}{\partial q_k}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{y}_i\frac{\partial y_i}{\partial q_k}\right]=F_i^y\frac{\partial y_i}{\partial q_k}+m_i\dot{y}_i\frac{\partial \dot{y}_i}{\partial q_k}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m_i\dot{z}_i\frac{\partial z_i}{\partial q_k}\right]=F_i^z\frac{\partial z_i}{\partial q_k}+m_i\dot{z}_i\frac{\partial \dot{z}_i}{\partial q_k}$$

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مکانیک لاگرانژی

ذره‌ی iام

ساده سازی

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)\right] = F_i^x\frac{\partial x_i}{\partial q_k} + m_i\dot{x}_i\frac{\partial \dot{x}_i}{\partial q_k}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)\right] = F_i^y\frac{\partial y_i}{\partial q_k} + m_i\dot{y}_i\frac{\partial \dot{y}_i}{\partial q_k}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)\right] = F_i^z\frac{\partial z_i}{\partial q_k} + m_i\dot{z}_i\frac{\partial \dot{z}_i}{\partial q_k}$$

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ذره‌ی iام

ساده سازی

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)\right] = F_i^x\frac{\partial x_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)\right] = F_i^y\frac{\partial y_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)\right] = F_i^z\frac{\partial z_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)$$

مکانیک لاگرانژی

ذره‌ی i ام

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)\right] = F_i^x\frac{\partial x_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{x}_i^2\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)\right] = F_i^y\frac{\partial y_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{y}_i^2\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right)\right] = F_i^z\frac{\partial z_i}{\partial q_k} + \frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{z}_i^2\right) \quad +$$

سمت راست $=$ سمت چپ

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مکانیک لاگرانژی

ذره‌ی i ام

سمت چپ $= \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\dfrac{\partial}{\partial \dot{q}_k}\left(\dfrac{1}{2}m_i\dot{x}_i^2\right)+\dfrac{\partial}{\partial \dot{q}_k}\left(\dfrac{1}{2}m_i\dot{y}_i^2\right)\dfrac{\partial}{\partial \dot{q}_k}\left(\dfrac{1}{2}m_i\dot{z}_i^2\right)\right]$

$= \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\dfrac{\partial}{\partial \dot{q}_k}\left(\dfrac{1}{2}m_i\dot{x}_i^2+\dfrac{1}{2}m_i\dot{y}_i^2+\dfrac{1}{2}m_i\dot{z}_i^2\right)\right]$

سمت راست $= \left(F_i^x\dfrac{\partial x_i}{\partial q_k}+F_i^y\dfrac{\partial y_i}{\partial q_k}+F_i^z\dfrac{\partial z_i}{\partial q_k}\right)$

$+\dfrac{\partial}{\partial q_k}\left(\dfrac{1}{2}m_i\dot{x}_i^2+\dfrac{1}{2}m_i\dot{y}_i^2+\dfrac{1}{2}m_i\dot{z}_i^2\right)$

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مکانیک لاگرانژی

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)\right]$$

$$=\left(F_i^x\frac{\partial x_i}{\partial q_k}+F_i^y\frac{\partial y_i}{\partial q_k}+F_i^z\frac{\partial z_i}{\partial q_k}\right)$$

$$+\frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)$$

مکانیک لاگرانژی

همه ذرات سیستم

$$\sum_i^N \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)\right]$$

$$=\sum_i^N\left(F_i^x\frac{\partial x_i}{\partial q_k}+F_i^y\frac{\partial y_i}{\partial q_k}+F_i^z\frac{\partial z_i}{\partial q_k}\right)$$

$$+\sum_i^N\frac{\partial}{\partial q_k}\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)$$

مکانیک لاگرانژی

همه ذرات سیستم

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\sum_i^N\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)\right]$$

$$=\sum_i^N\left(F_i^x\frac{\partial x_i}{\partial q_k}+F_i^y\frac{\partial y_i}{\partial q_k}+F_i^z\frac{\partial z_i}{\partial q_k}\right)$$

$$+\frac{\partial}{\partial q_k}\sum_i^N\left(\frac{1}{2}m_i\dot{x}_i^2+\frac{1}{2}m_i\dot{y}_i^2+\frac{1}{2}m_i\dot{z}_i^2\right)$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_i^N \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = \sum_i^N \left(F_i^x \frac{\partial x_i}{\partial q_k} + F_i^y \frac{\partial y_i}{\partial q_k} + F_i^z \frac{\partial z_i}{\partial q_k} \right)$$
مختصه‌ی kام

$$\frac{\mathrm{d}}{\mathrm{d}t} \left[\frac{\partial T}{\partial \dot{q}_k} \right] = Q_k + \frac{\partial T}{\partial q_k}, \quad 1 \leq k \leq n$$

معادلات لاگرانژ (در فُرم انرژی جنبشی)

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_{i}^{N} \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = \sum_{i}^{N} \left(F_i^x \frac{\partial x_i}{\partial q_k} + F_i^y \frac{\partial y_i}{\partial q_k} + F_i^z \frac{\partial z_i}{\partial q_k} \right)$$

مختصه‌ی kام

$$V = V(x_1, y_1, z_1, x_2, y_2, z_2, \cdots, x_i, y_i, z_i, \cdots, x_N, y_N, z_N)$$

$$F_i^x = -\frac{\partial V}{\partial x_i}, \; F_i^y = -\frac{\partial V}{\partial y_i}, \; F_i^z = -\frac{\partial V}{\partial z_i}$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_i^N \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = \sum_i^N \left(F_i^x \frac{\partial x_i}{\partial q_k} + F_i^y \frac{\partial y_i}{\partial q_k} + F_i^z \frac{\partial z_i}{\partial q_k} \right)$$

مختصه‌ی kام

$$V = V(x_1, y_1, z_1, x_2, y_2, z_2, \cdots, x_i, y_i, z_i, \cdots, x_N, y_N, z_N)$$

$$Q_k = -\sum_i^N \left(\frac{\partial V}{\partial x_i} \frac{\partial x_i}{\partial q_k} + \frac{\partial V}{\partial y_i} \frac{\partial y_i}{\partial q_k} + \frac{\partial V}{\partial z_i} \frac{\partial z_i}{\partial q_k} \right) = -\frac{\partial V}{\partial q_k}$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_{i}^{N} \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = -\frac{\partial V}{\partial q_k}$$
مختصه‌ی kام

$$V = V(q_1, q_2, \cdots, q_k, \cdots, q_n)$$

$$\frac{\partial V}{\partial \dot{q}_k} = 0$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_{i}^{N} \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = -\frac{\partial V}{\partial q_k}, \qquad \frac{\partial V}{\partial \dot{q}_k} = 0$$

مختصه‌ی kام

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial T}{\partial \dot{q}_k}\right] = Q_k + \frac{\partial T}{\partial q_k} = -\frac{\partial V}{\partial q_k} + \frac{\partial T}{\partial q_k}, \qquad 1 \leq k \leq n$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial T}{\partial \dot{q}_k}\right] = \frac{\partial}{\partial q_k}(T - V)$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_i^N \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = -\frac{\partial V}{\partial q_k}, \quad \frac{\partial V}{\partial \dot{q}_k} = 0$$

مختصه‌ی kام

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial T}{\partial \dot{q}_k}\right] = \frac{\partial}{\partial q_k}(T - V) \Rightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial V}{\partial \dot{q}_k}\right] = \frac{\partial}{\partial q_k}(T - V)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}(T - V)\right] = \frac{\partial}{\partial q_k}(T - V)$$

مکانیک لاگرانژی

همه ذرات سیستم

انرژی جنبش کل : $$T = \sum_{i}^{N} \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : $$Q_k = -\frac{\partial V}{\partial q_k}, \quad \frac{\partial V}{\partial \dot{q}_k} = 0$$

مختصه‌ی kام

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}(T - V)\right] = \frac{\partial}{\partial q_k}(T - V), \quad \mathcal{L} = T - V$$

معادلات لاگرانژ:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial}{\partial \dot{q}_k}\mathcal{L}\right] = \frac{\partial}{\partial q_k}\mathcal{L}, \quad 1 \leq k \leq n}$$

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مکانیک لاگرانژی

خلاصه بحث:

انرژی جنبش کل :
$$T = \sum_i^N \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

نیروی تعمیم یافته : kامین مختصه‌ی
$$Q_k = \sum_i^N \left(F_i^x \frac{\partial x_i}{\partial q_k} + F_i^y \frac{\partial y_i}{\partial q_k} + F_i^z \frac{\partial z_i}{\partial q_k} \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left[\frac{\partial T}{\partial \dot{q}_k} \right] = Q_k + \frac{\partial T}{\partial q_k}, \quad 1 \leq k \leq n$$

معادلات لاگرانژ (در فُرم انرژی جنبشی)

مکانیک لاگرانژی

خلاصه بحث:

انرژی جنبش کل :
$$T = \sum_i^N \left(\frac{1}{2} m_i \dot{x}_i^2 + \frac{1}{2} m_i \dot{y}_i^2 + \frac{1}{2} m_i \dot{z}_i^2 \right)$$

انرژی پتانسیل کل :
$$V = V(q_1, q_2, \cdots, q_k, \cdots, q_n)$$

$$\mathcal{L} = T - V$$

معادلات لاگرانژ:
$$\frac{\mathrm{d}}{\mathrm{d}t} \left[\frac{\partial}{\partial \dot{q}_k} \mathcal{L} \right] = \frac{\partial}{\partial q_k} \mathcal{L}, \quad 1 \leq k \leq n$$