جلسه هفدهم

مکانیک تحلیلی

محمدرضا مظفری
گروه فیزیک، دانشکده علوم پایه
دانشگاه قم
اسفند ۹۸

مکانیک لاگرانژی

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = 0, \quad i = 1, 2, \cdots, n$$

قیدها هولونومیک $f_k = f_k(\{q\}), \quad k = 1, 2, \cdots, M$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = \frac{\partial \mathcal{L}}{\partial q_i} + \sum_k \lambda_k \frac{\partial f_k}{\partial q_i}, \quad i = 1, 2, \cdots, n}$$

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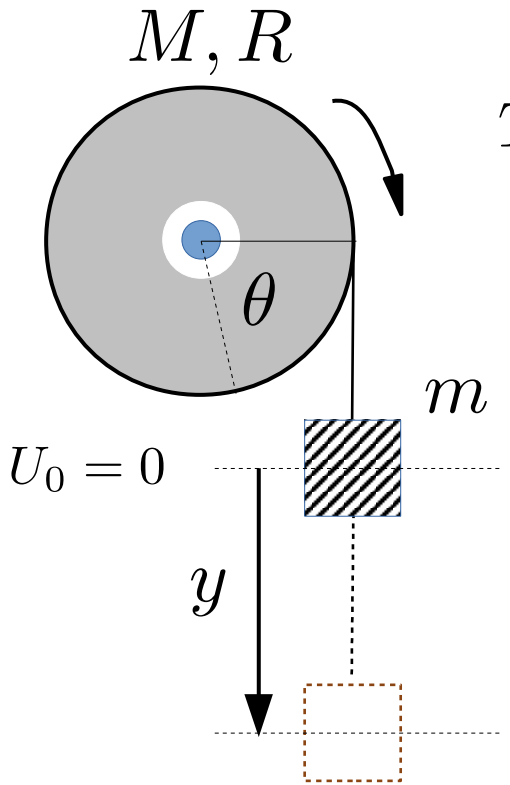

$$T = \frac{1}{2}\mathbb{I}\dot{\theta}^2 + \frac{1}{2}m\dot{y}^2 = \frac{1}{4}MR^2\dot{\theta}^2 + \frac{1}{2}m\dot{y}^2, \quad V = -mgy$$

$$f = y - R\theta$$

$$\mathcal{L} = \frac{1}{4}MR^2\dot{\theta}^2 + \frac{1}{2}m\dot{y}^2 + mgy$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial \mathcal{L}}{\partial \dot{y}}\right] = \frac{\partial \mathcal{L}}{\partial y} + \lambda\frac{\partial f}{\partial y} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right] = \frac{\partial \mathcal{L}}{\partial \theta} + \lambda\frac{\partial f}{\partial \theta}$$

$$m\ddot{y} = mg + \lambda \qquad \frac{1}{2}MR^2\ddot{\theta} = -R\lambda$$

$$f = y - R\theta = 0 \Rightarrow \dot{y} = R\dot{\theta} \Rightarrow \ddot{y} = R\ddot{\theta}$$

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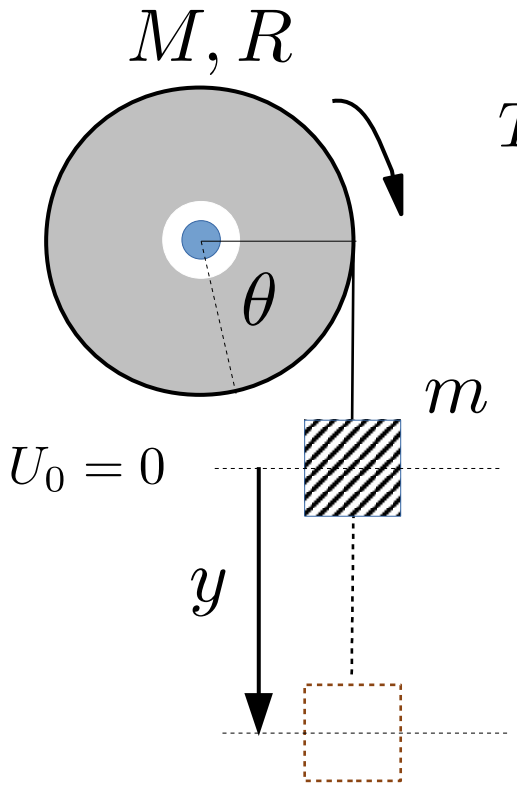

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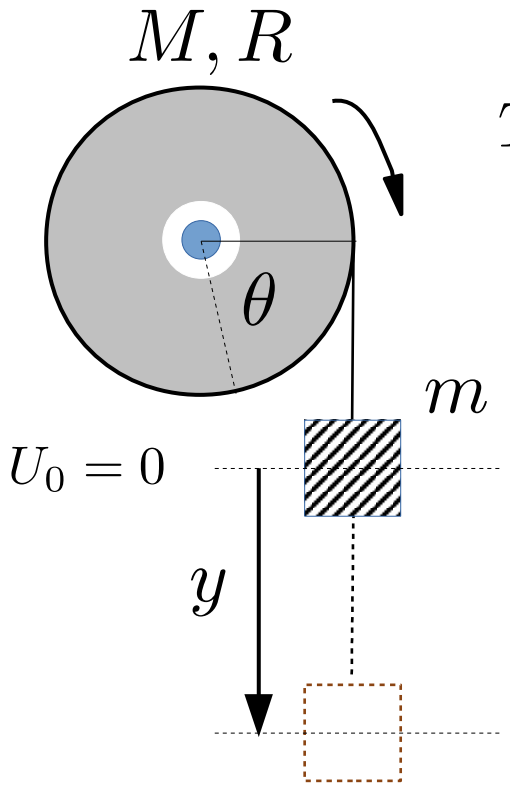

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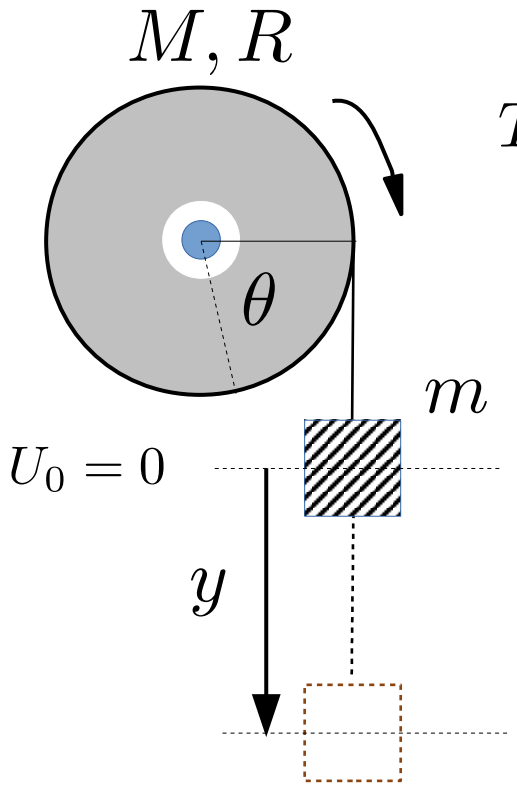

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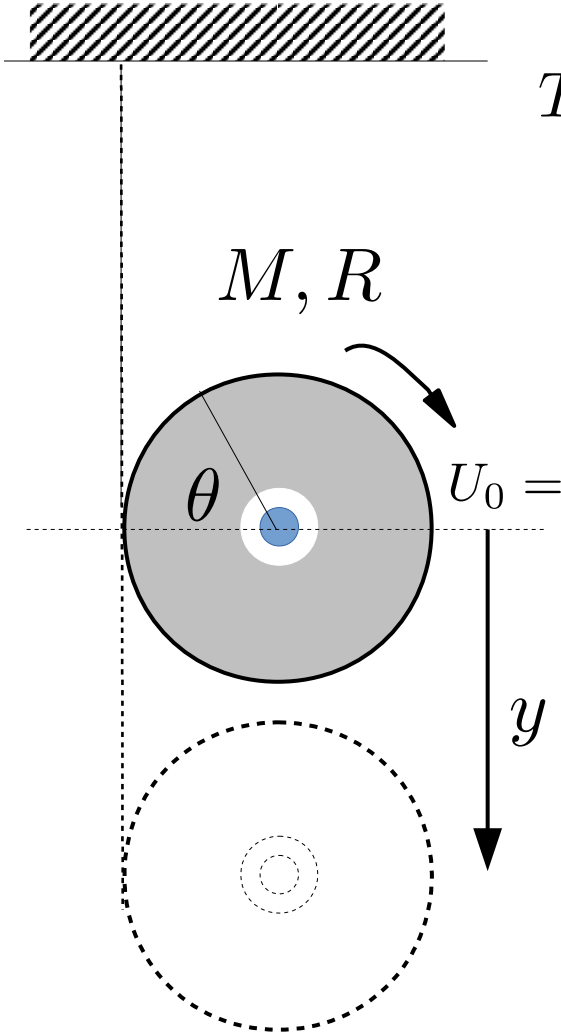

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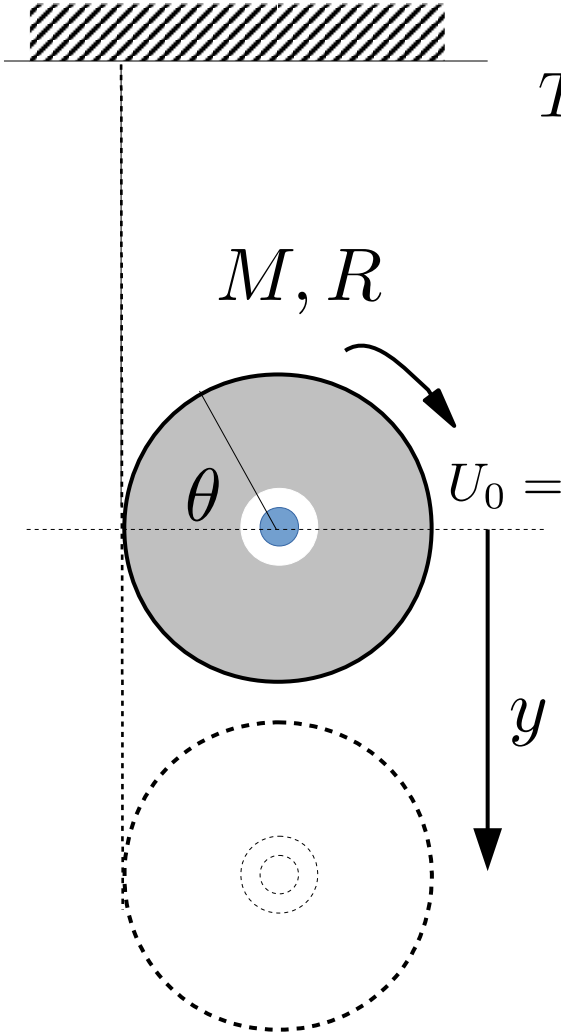

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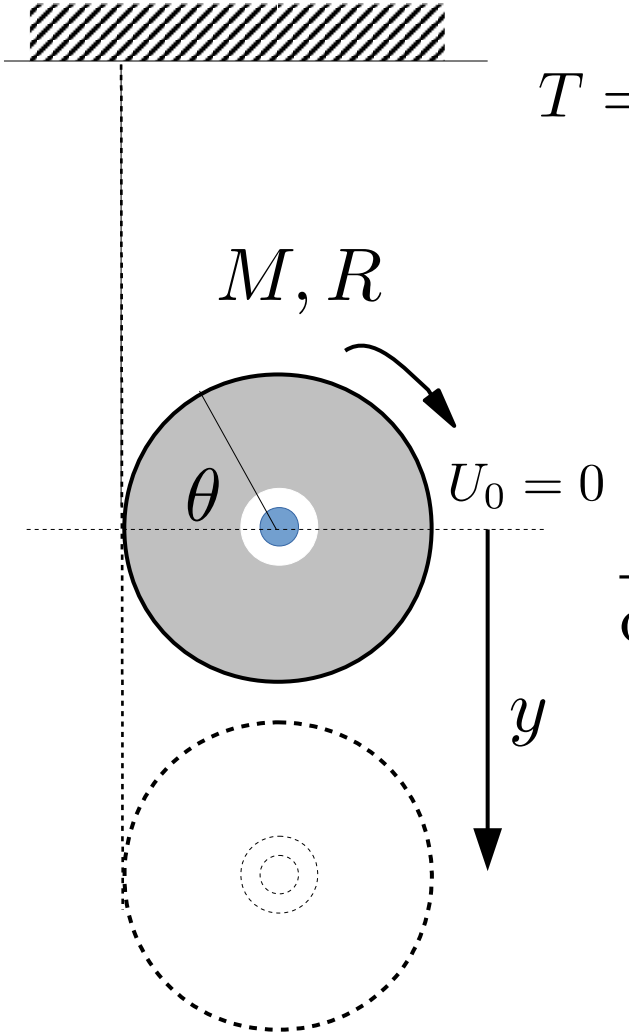

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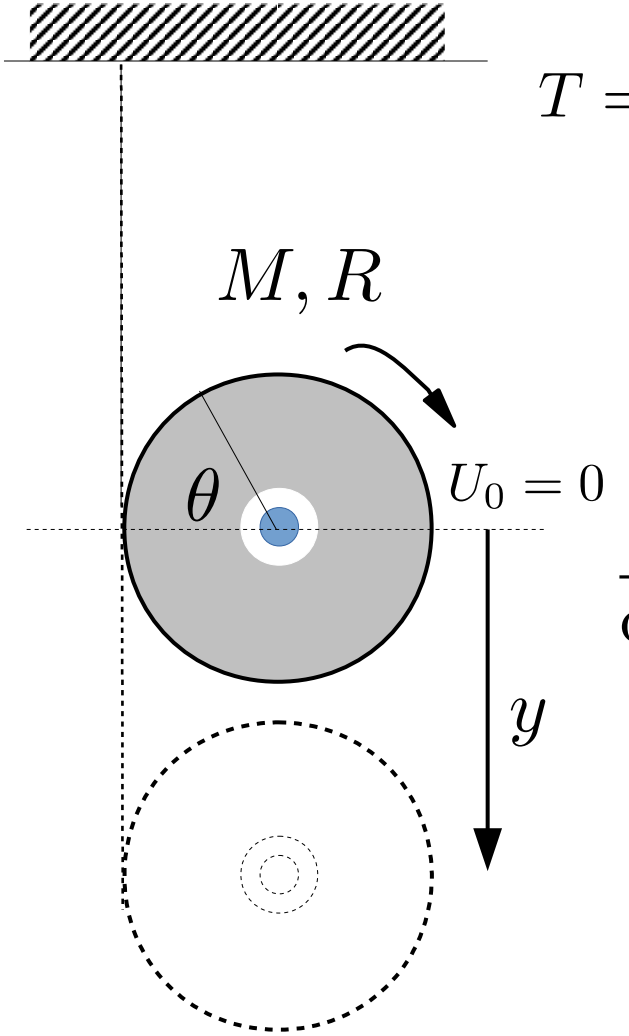

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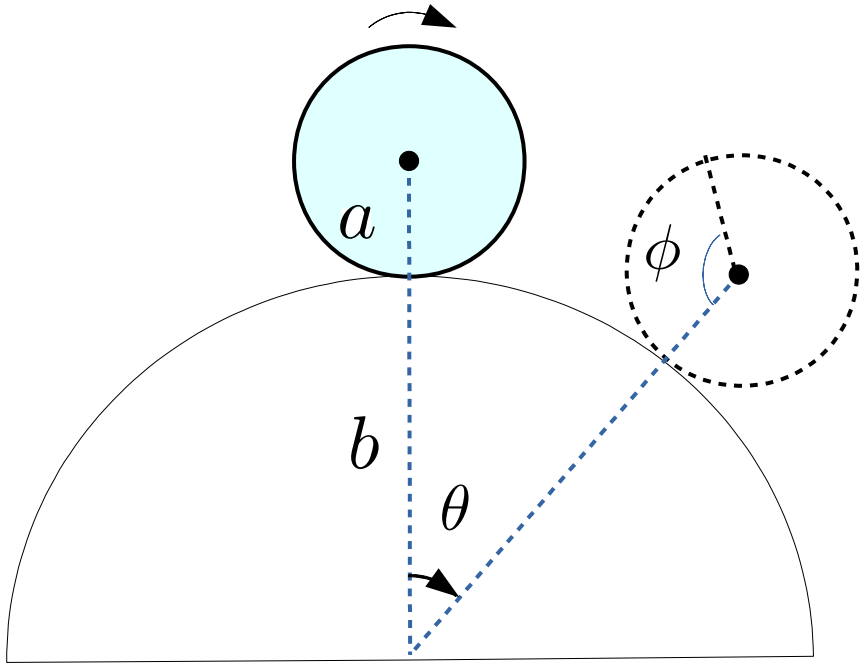

$$\begin{cases} x = (a+r)\sin\theta \\ y = (a+r)\cos\theta \end{cases}$$

$$\begin{cases} \dot{x} = \dot{r}\sin\theta + (a+r)\dot{\theta}\cos\theta \\ \dot{y} = \dot{r}\cos\theta + -(r+b)\dot{\theta}\sin\theta \end{cases}$$

$$f_1 = (a+b)\theta - a\phi, \qquad f_2 = r - b$$

$$T = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{2}\mathbb{I}_o\dot{\phi}^2$$

$$T = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{2}Ma^2\right)\dot{\phi}^2, \quad V = -mg(a+r)\cos\theta$$

$$\mathcal{L} = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{4}Ma^2\dot{\phi}^2 - Mg(a+r)\cos\theta$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right] = \frac{\partial\mathcal{L}}{\partial\theta} + \lambda_1\frac{\partial f_1}{\partial\theta}$$

$$M(a+r)^2\ddot{\theta} + 2M(a+r)\dot{r}\dot{\theta} = Mg(a+r)\sin\theta + (a+b)\lambda_1$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right] = \frac{\partial\mathcal{L}}{\partial\phi} + \lambda_1\frac{\partial f_1}{\partial\phi}$$

$$\frac{1}{2}Ma^2\ddot{\phi} = -a\lambda_1$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{r}}\right] = \frac{\partial\mathcal{L}}{\partial r} + \lambda_2\frac{\partial f_2}{\partial r}$$

$$M\ddot{r} = M(a+r)\dot{\theta}^2 - Mg\cos\theta + \lambda_2$$

$$f_1 = (a+b)\theta - a\phi = 0$$

$$(a+b)\dot{\theta} = a\dot{\phi} \Rightarrow (a+b)\ddot{\theta} = a\ddot{\phi}$$

$$f_2 = r - b = 0$$

$$\dot{r} = \ddot{r} = 0$$

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$$\mathcal{L} = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{4}Ma^2\dot{\phi}^2 - Mg(a+r)\cos\theta$$

$$(a+b)\dot{\theta} = a\dot{\phi}, \quad (a+b)\ddot{\theta} = a\ddot{\phi}, \quad r = b, \quad \dot{r} = \ddot{r} = 0$$

$$\begin{cases} M(a+b)\ddot{\theta} + 2M\dot{r}\dot{\theta} = Mg\sin\theta + \lambda_1 \quad (\dot{r} \to 0) \\ \frac{1}{2}Ma\ddot{\phi} = -\lambda_1 \\ M\ddot{r} = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \quad (\ddot{r} \to 0) \end{cases}$$

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$$0 = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \Rightarrow \lambda_2 = -M(a+b)\dot{\theta}^2 + Mg\cos\theta$$

$$\ddot{\theta} = \frac{2g}{3(a+b)}\sin\theta$$

$$\dot{\theta}\mathrm{d}\dot{\theta} = \frac{2g}{3(a+b)}\sin\theta\mathrm{d}\theta \Rightarrow \int_0^{\dot{\theta}} \dot{\theta}\mathrm{d}\dot{\theta} = \frac{2g}{3(a+b)}\int_0^{\theta}\sin\theta\mathrm{d}\theta$$

$$\frac{1}{2}\dot{\theta}^2 = \frac{2g}{3(a+b)}(1-\cos\theta)$$

$$N = \lambda_2 = -\frac{4}{3}Mg(1-\cos\theta) + Mg\cos\theta = -\frac{4}{3}Mg + \frac{7}{3}Mg\cos\theta$$

$$N = 0 \Rightarrow \cos\theta = \frac{4}{7} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{7}\right)$$

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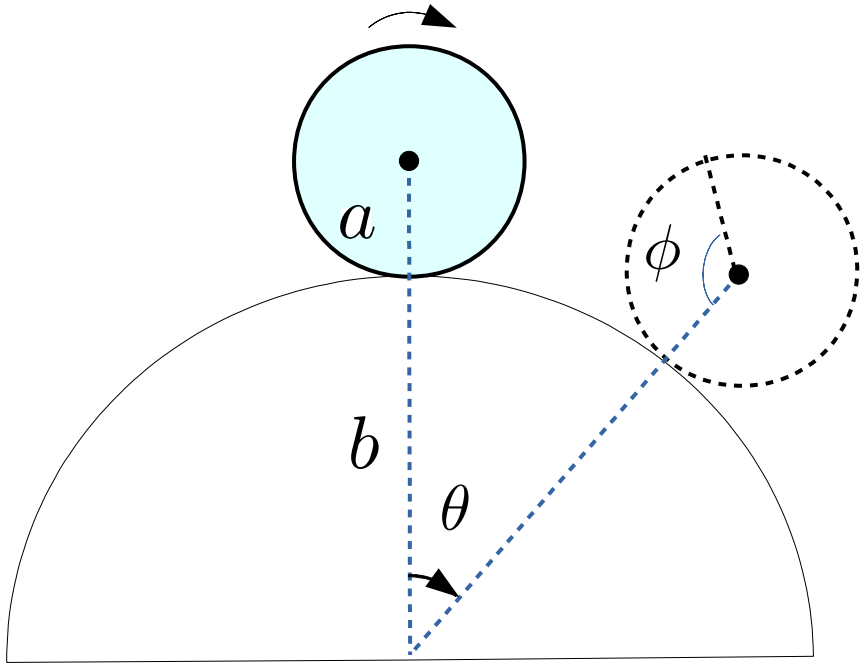

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$$(a+b)\ddot{\theta} = a\ddot{\phi}$$

$$f_2 = r - b = 0$$

$$\dot{r} = \ddot{r} = 0$$

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$$\mathcal{L} = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{4}Ma^2\dot{\phi}^2 - Mg(a+r)\cos\theta$$

$$(a+b)\dot{\theta} = a\dot{\phi}, \quad (a+b)\ddot{\theta} = a\ddot{\phi}, \quad r = b, \quad \dot{r} = \ddot{r} = 0$$

$$\begin{cases} M(a+b)\ddot{\theta} + 2M\dot{r}\dot{\theta} = Mg\sin\theta + \lambda_1 \\ \dfrac{1}{2}Ma\ddot{\phi} = -\lambda_1 \\ M\ddot{r} = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \end{cases}$$

$$\begin{cases} M(a+b)\ddot{\theta} = Mg\sin\theta + \lambda_1 \\ \dfrac{1}{2}M(a+b)\ddot{\theta} = -\lambda_1 \\ 0 = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \end{cases}$$

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$$\mathcal{L} = \frac{1}{2}M\dot{r}^2 + \frac{1}{2}M(a+r)^2\dot{\theta}^2 + \frac{1}{4}Ma^2\dot{\phi}^2 - Mg(a+r)\cos\theta$$

$$(a+b)\dot{\theta} = a\dot{\phi}, \quad (a+b)\ddot{\theta} = a\ddot{\phi}, \quad r = b, \quad \dot{r} = \ddot{r} = 0$$

$$\begin{cases} M(a+b)\ddot{\theta} = Mg\sin\theta + \lambda_1 \\ \frac{1}{2}M(a+b)\ddot{\theta} = -\lambda_1 \\ 0 = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \end{cases}$$

$$\begin{cases} M(a+b)\ddot{\theta} = Mg\sin\theta + \lambda_1 \\ \frac{1}{2}M(a+b)\ddot{\theta} = -\lambda_1 \end{cases} \Rightarrow \begin{cases} \ddot{\theta} = \frac{2g}{3(a+b)}\sin\theta \\ \lambda_1 = -\frac{1}{3}Mg\sin\theta \end{cases}$$

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$$0 = M(a+b)\dot{\theta}^2 - Mg\cos\theta + \lambda_2 \Rightarrow \lambda_2 = -M(a+b)\dot{\theta}^2 + Mg\cos\theta$$

$$\ddot{\theta} = \frac{2g}{3(a+b)}\sin\theta$$

$$\dot{\theta}\mathrm{d}\dot{\theta} = \frac{2g}{3(a+b)}\sin\theta\mathrm{d}\theta \Rightarrow \int_0^{\dot{\theta}} \dot{\theta}\mathrm{d}\dot{\theta} = \frac{2g}{3(a+b)}\int_0^{\theta}\sin\theta\mathrm{d}\theta$$

$$\frac{1}{2}\dot{\theta}^2 = \frac{2g}{3(a+b)}(1-\cos\theta)$$

$$N = \lambda_2 = -\frac{4}{3}Mg(1-\cos\theta) + Mg\cos\theta = -\frac{4}{3}Mg + \frac{7}{3}Mg\cos\theta$$

$$N = 0 \Rightarrow \cos\theta = \frac{4}{7} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{7}\right)$$

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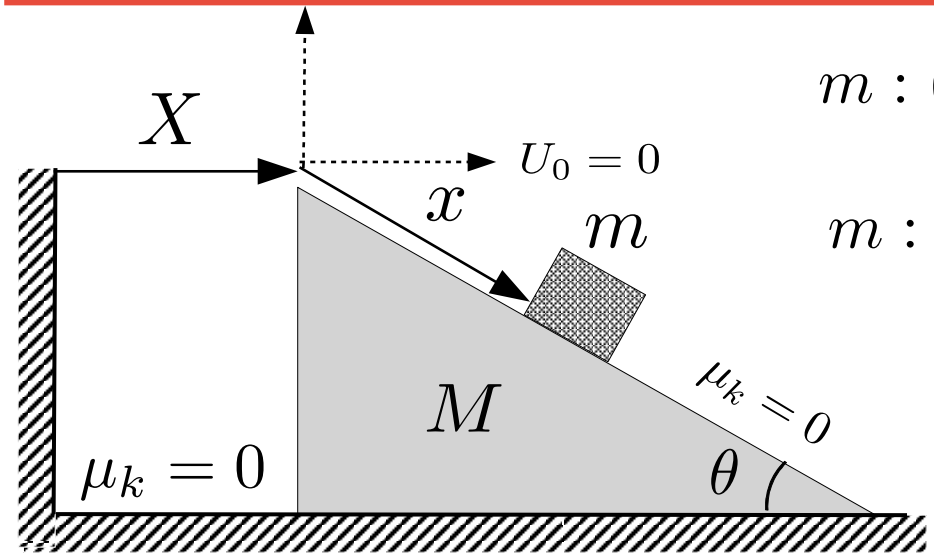

$$m : (X + x\cos\theta, -x\sin\theta)$$

$$m : (\dot{X} + \dot{x}\cos\theta - x\dot{\theta}\sin\theta, -\dot{x}\sin\theta - x\dot{\theta}\cos\theta)$$

$$M : (X, 0)$$

$$M : (\dot{X}, 0)$$

$$f(\theta) = \theta - \theta_0$$

$$T = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta)$$

$$V = -mgx\sin\theta$$

$$\mathcal{L} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta) + mgx\sin\theta$$

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$$\mathcal{L} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta)$$
$$+mgx\sin\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{X}}\right] = \frac{\partial\mathcal{L}}{\partial X} + \lambda\frac{\partial f}{\partial X}$$

$$(M+m)\ddot{X} + m\ddot{x}\cos\theta - 2m\dot{x}\dot{\theta}\sin\theta - mx\ddot{\theta}\sin\theta - mx\dot{\theta}^2\cos\theta = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{x}}\right] = \frac{\partial\mathcal{L}}{\partial x} + \lambda\frac{\partial f}{\partial x}$$

$$m\ddot{x} + m\ddot{X}\cos\theta - m\dot{X}\dot{\theta}\sin\theta = mx\dot{\theta}^2 - m\dot{X}\dot{\theta}\sin\theta + mg\sin\theta$$

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$$\mathcal{L} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta)$$

$$+mgx\sin\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right] = \frac{\partial\mathcal{L}}{\partial\theta} + \lambda\frac{\partial f}{\partial\theta}$$

$$mx^2\ddot{\theta} + 2x\dot{x}\dot{\theta} - m\ddot{X}x\sin\theta - m\dot{X}\dot{x}\sin\theta - m\dot{X}x\dot{\theta}\cos\theta$$

$$= -m\dot{X}\dot{x}\sin\theta - m\dot{X}x\dot{\theta}\cos\theta + mgx\cos\theta + \lambda$$

$$mx^2\ddot{\theta} + 2x\dot{x}\dot{\theta} - m\ddot{X}x\sin\theta = mgx\cos\theta + \lambda$$

$$f(\theta) = \theta - \theta_0 = 0 \Rightarrow \theta = \theta_0 \Rightarrow \ddot{\theta} = \dot{\theta} = 0$$

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$$\mathcal{L} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta) + mgx\sin\theta$$

$$f(\theta) = \theta - \theta_0 = 0 \Rightarrow \theta = \theta_0 \Rightarrow \ddot{\theta} = \dot{\theta} = 0$$

$$(M+m)\ddot{X} + m\ddot{x}\cos\theta - 2m\dot{x}\cancelto{0}{\dot{\theta}}\sin\theta - mx\cancelto{0}{\ddot{\theta}}\sin\theta - mx\cancelto{0}{\dot{\theta}^2}\cos\theta = 0$$

$$m\ddot{x} + m\ddot{X}\cos\theta - m\dot{X}\cancelto{0}{\dot{\theta}}\sin\theta = mx\cancelto{0}{\dot{\theta}^2} - m\dot{X}\cancelto{0}{\dot{\theta}}\sin\theta + mg\sin\theta$$

$$mx^2\cancelto{0}{\ddot{\theta}} + 2x\dot{x}\cancelto{0}{\dot{\theta}} - m\ddot{X}x\sin\theta = mgx\cos\theta + \lambda$$

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$$\mathcal{L} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}m(\dot{X}^2 + \dot{x}^2 + x^2\dot{\theta}^2 + 2\dot{X}\dot{x}\cos\theta - 2\dot{X}x\dot{\theta}\sin\theta) + mgx\sin\theta$$

$$f(\theta) = \theta - \theta_0 = 0 \Rightarrow \theta = \theta_0 \Rightarrow \ddot{\theta} = \dot{\theta} = 0$$

$$\left.\begin{aligned}(M+m)\ddot{X} + m\ddot{x}\cos\theta_0 = 0 \\ m\ddot{x} + m\ddot{X}\cos\theta_0 = mg\sin\theta_0\end{aligned}\right\} \Rightarrow \begin{aligned}\ddot{x} &= \frac{(M+m)g\sin\theta_0}{M+m-m\cos^2\theta_0} \\ \ddot{X} &= -\frac{mg\sin\theta_0\cos\theta_0}{M+m-m\cos^2\theta_0}\end{aligned}$$

$$-m\ddot{X}x\sin\theta_0 = mgx\cos\theta_0 + \lambda \Rightarrow \lambda = -mx(\ddot{X}\sin\theta_0 + g\cos\theta_0)$$

$$\lambda = -x\left(\frac{mMg\cos\theta_0}{M+m-m\cos^2\theta_0}\right) = -xN$$

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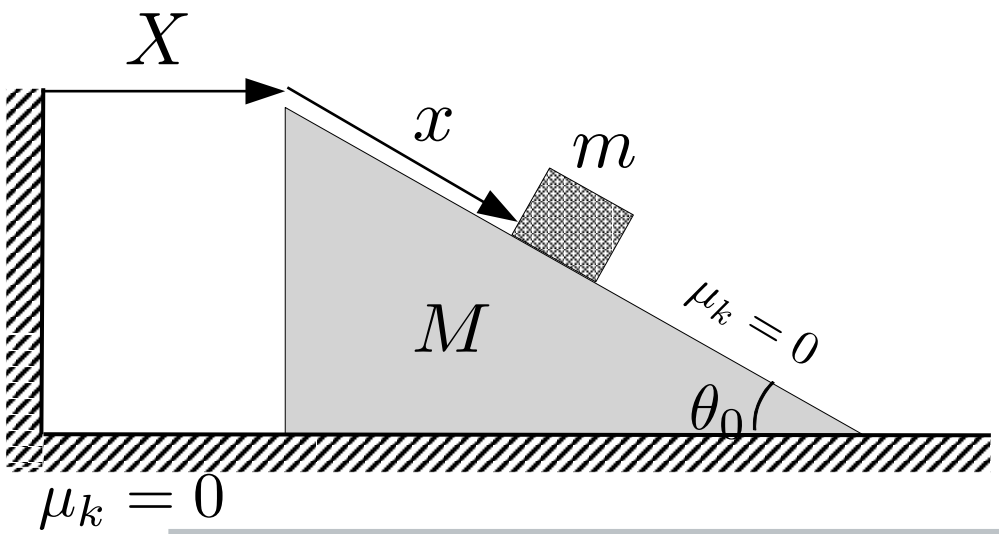
$$\ddot{x} = \frac{(M+m)g\sin\theta_0}{M+m-m\cos^2\theta_0} \qquad \lim_{\frac{M}{m}\to\infty} \ddot{x} = g\sin\theta_0$$

$$\ddot{X} = -\frac{mg\sin\theta_0\cos\theta_0}{M+m-m\cos^2\theta_0} \qquad \lim_{\frac{M}{m}\to\infty} \ddot{X} = 0$$



$$N = \left(\frac{mMg\cos\theta_0}{M+m-m\cos^2\theta_0}\right)$$

$$\lim_{\frac{M}{m}\to\infty} N = mg\cos\theta_0$$

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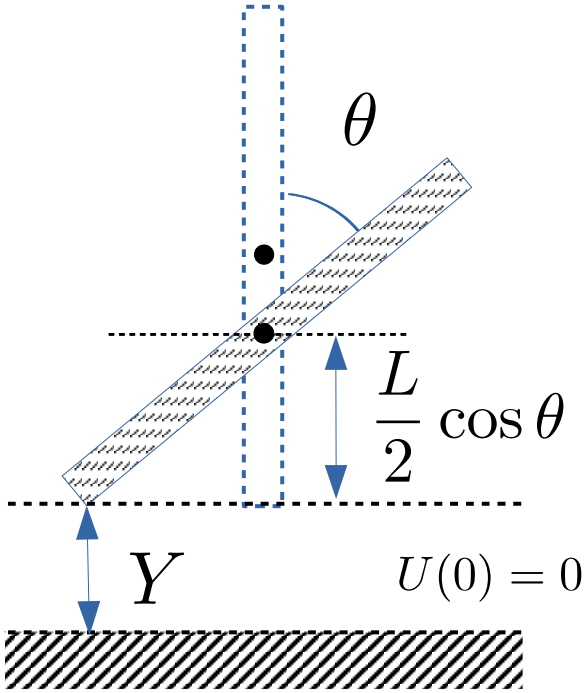

$$y = Y + \frac{L}{2}\cos\theta, \qquad \dot{y} = \dot{Y} - \frac{L}{2}\dot{\theta}\sin\theta$$

$$T = \frac{1}{2}M\dot{y}^2 + \frac{1}{2}\mathbb{I}_{\mathrm{CM}}\dot{\theta}^2$$

$$T = \frac{1}{2}M\left(\dot{Y} - \frac{L}{2}\dot{\theta}\sin\theta\right)^2 + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\dot{\theta}^2$$

$$V = Mg\left(Y + \frac{L}{2}\cos\theta\right), \qquad f = Y$$

$$\mathcal{L} = \frac{1}{2}M\dot{Y}^2 - \frac{1}{2}M\dot{Y}L\dot{\theta}\sin\theta + \frac{1}{8}ML^2\dot{\theta}^2\sin^2\theta + \frac{1}{24}ML^2\dot{\theta}^2$$

$$-Mg\left(Y + \frac{L}{2}\cos\theta\right)$$

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$$\mathcal{L} = \frac{1}{2}M\dot{Y}^2 - \frac{1}{2}M\dot{Y}L\dot{\theta}\sin\theta + \frac{1}{8}ML^2\dot{\theta}^2\sin^2\theta + \frac{1}{24}ML^2\dot{\theta}^2 - Mg\left(Y + \frac{L}{2}\cos\theta\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{Y}}\right] = \frac{\partial\mathcal{L}}{\partial Y} + \lambda\frac{\partial f}{\partial Y}$$

$$M\ddot{Y} - \frac{1}{2}ML\ddot{\theta}\sin\theta - \frac{1}{2}ML\dot{\theta}^2\cos\theta = -Mg + \lambda$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right] = \frac{\partial\mathcal{L}}{\partial\theta} + \lambda\frac{\partial f}{\partial\theta}$$

$$-\frac{1}{2}M\ddot{Y}L\sin\theta - \frac{1}{2}M\dot{Y}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\ddot{\theta}\sin^2\theta + \frac{1}{2}ML^2\dot{\theta}^2\sin\theta\cos\theta$$

$$\frac{1}{12}ML^2\ddot{\theta} = -\frac{1}{2}M\dot{Y}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\dot{\theta}^2\sin\theta\cos\theta + Mg\frac{L}{2}\sin\theta$$

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$$M\ddot{Y} - \frac{1}{2}ML\ddot{\theta}\sin\theta - \frac{1}{2}ML\dot{\theta}^2\cos\theta = -Mg + \lambda$$

$$-\frac{1}{2}M\ddot{Y}L\sin\theta - \frac{1}{2}M\dot{Y}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\ddot{\theta}\sin^2\theta + \frac{1}{2}ML^2\dot{\theta}^2\sin\theta\cos\theta$$

$$\frac{1}{12}ML^2\ddot{\theta} = -\frac{1}{2}M\dot{Y}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\dot{\theta}^2\sin\theta\cos\theta + Mg\frac{L}{2}\sin\theta$$

$$f = Y = 0 \Rightarrow \dot{Y} = \ddot{Y} = 0$$

$$\cancel{M\ddot{Y}}^{0} - \frac{1}{2}ML\ddot{\theta}\sin\theta - \frac{1}{2}ML\dot{\theta}^2\cos\theta = -Mg + \lambda$$

$$-\frac{1}{2}\cancel{M\ddot{Y}}^{0}L\sin\theta - \frac{1}{2}\cancel{M\dot{Y}}^{0}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\ddot{\theta}\sin^2\theta + \frac{1}{2}ML^2\dot{\theta}^2\sin\theta\cos\theta$$

$$\frac{1}{12}ML^2\ddot{\theta} = -\frac{1}{2}\cancel{M\dot{Y}}^{0}L\dot{\theta}\cos\theta + \frac{1}{4}ML^2\dot{\theta}^2\sin\theta\cos\theta + Mg\frac{L}{2}\sin\theta$$

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$$\begin{cases} -\frac{1}{2}ML\ddot{\theta}\sin\theta - \frac{1}{2}ML\dot{\theta}^2\cos\theta = -Mg + \lambda \\ \frac{1}{4}ML^2\ddot{\theta}\sin^2\theta + \frac{1}{2}ML^2\dot{\theta}^2\sin\theta\cos\theta + \frac{1}{12}ML^2\ddot{\theta} \\ \qquad = \frac{1}{4}ML^2\dot{\theta}^2\sin\theta\cos\theta + Mg\frac{L}{2}\sin\theta \end{cases}$$

$$\begin{cases} \lambda = Mg - \frac{1}{2}ML(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \\ \frac{1}{12}ML^2(1+3\sin^2\theta)\ddot{\theta} + \frac{1}{4}ML^2\dot{\theta}^2\sin\theta\cos\theta = Mg\frac{L}{2}\sin\theta \end{cases}$$

$$\begin{cases} \lambda = Mg - \frac{1}{2}ML(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \\ (1+3\sin^2\theta)\ddot{\theta} + 3\dot{\theta}^2\sin\theta\cos\theta = \frac{6g}{L}\sin\theta \end{cases}$$

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$$(1+3\sin^2\theta)\ddot{\theta}+3\dot{\theta}^2\sin\theta\cos\theta=\frac{6g}{L}\sin\theta$$

$$(1+3\sin^2\theta)\ddot{\theta}+3\dot{\theta}^2\sin\theta\cos\theta=\frac{6g}{L}\sin\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\dot{\theta}\left[(1+3\sin^2\theta)\ddot{\theta}+3\dot{\theta}^2\sin\theta\cos\theta\right]$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\dot{\theta}\left[(1+3\sin^2\theta)\ddot{\theta}+3\dot{\theta}^2\sin\theta\cos\theta\right]$$

$$\cancel{\dot{\theta}}\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\cancel{\dot{\theta}}\left[(1+3\sin^2\theta)\ddot{\theta}+3\dot{\theta}^2\sin\theta\cos\theta\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\frac{6g}{L}\sin\theta$$

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$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\frac{6g}{L}\sin\theta$$

$$\int_{(0,0)}^{(\theta,\dot{\theta})}\mathrm{d}\left[\frac{1}{2}(1+3\sin^2\theta)\dot{\theta}^2\right]=\frac{6g}{L}\int_0^{\theta}\sin\theta\mathrm{d}\theta$$

$$(1+3\sin^2\theta)\dot{\theta}^2=\frac{12g}{L}(1-\cos\theta)\Rightarrow\dot{\theta}^2=\frac{12g}{L}\left(\frac{1-\cos\theta}{1+3\sin^2\theta}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\dot{\theta}^2=\frac{12g}{L}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1-\cos\theta}{1+3\sin^2\theta}\right)$$

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$$2\dot{\theta}\ddot{\theta} = \frac{12g}{L}\left[\frac{\sin\theta(1+3\sin^2\theta)-6(1-\cos\theta)\sin\theta\cos\theta}{(1+3\sin^2\theta)^2}\right]\dot{\theta}$$

$$\ddot{\theta} = \frac{6g}{L}\left[\frac{\sin\theta(1+3\sin^2\theta)-6(1-\cos\theta)\sin\theta\cos\theta}{(1+3\sin^2\theta)^2}\right]$$

$$\dot{\theta}^2 = 12\frac{g}{L}\left(\frac{1-\cos\theta}{1+3\sin^2\theta}\right)$$

$$\begin{cases} \lambda = Mg - \frac{1}{2}ML(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \\ (1+3\sin^2\theta)\ddot{\theta} + 3\dot{\theta}^2\sin\theta\cos\theta = \frac{6g}{L}\sin\theta \end{cases}$$

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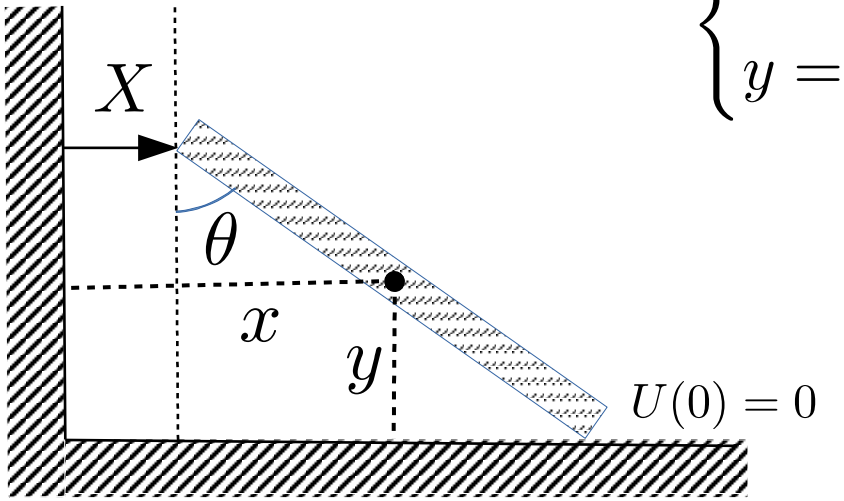

$$\begin{cases} x = X + \frac{L}{2}\sin\theta \\ y = \frac{L}{2}\cos\theta \end{cases} \qquad \begin{cases} \dot{x} = \dot{X} + \dot{\theta}\frac{L}{2}\cos\theta \\ \dot{y} = -\dot{\theta}\frac{L}{2}\sin\theta \end{cases}$$

$$V = Mg\frac{L}{2}\cos\theta$$

$$f = X$$

$$T = \frac{1}{2}M\left(\frac{1}{4}L^2\dot{\theta}^2 + L\dot{\theta}\dot{X}\cos\theta + \dot{X}^2\right) + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\dot{\theta}^2$$

$$\mathcal{L} = \frac{1}{2}M\left(\frac{1}{4}L^2\dot{\theta}^2 + L\dot{\theta}\dot{X}\cos\theta + \dot{X}^2\right) + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\dot{\theta}^2 - Mg\frac{L}{2}\cos\theta$$

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$$\mathcal{L} = \frac{1}{2}M\left(\frac{1}{4}L^2\dot{\theta}^2 + L\dot{\theta}\dot{X}\cos\theta + \dot{X}^2\right) + \frac{1}{2}\left(\frac{1}{12}ML^2\right)\dot{\theta}^2 - Mg\frac{L}{2}\cos\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right] = \frac{\partial\mathcal{L}}{\partial\theta} + \lambda\frac{\partial f}{\partial\theta}$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{X}\cos\theta - \frac{1}{2}ML\dot{\theta}\dot{X}\sin\theta = -\frac{1}{2}ML\dot{\theta}\dot{X}\sin\theta + Mg\frac{L}{2}\sin\theta$$

$$\frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{X}\cos\theta = Mg\frac{L}{2}\sin\theta$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial\mathcal{L}}{\partial\dot{X}}\right] = \frac{\partial\mathcal{L}}{\partial X} + \lambda\frac{\partial f}{\partial X}$$

$$M\ddot{X} + \frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\sin\theta = \lambda$$

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$$\begin{cases} \frac{1}{3}ML^2\ddot{\theta} + \frac{1}{2}ML\ddot{X}\cos\theta = Mg\frac{L}{2}\sin\theta \\ M\ddot{X} + \frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\sin\theta = \lambda \end{cases}$$

$$f = X = 0 \Rightarrow \dot{X} = \ddot{X} = 0$$

$$\begin{cases} \frac{1}{3}ML^2\ddot{\theta} = Mg\frac{L}{2}\sin\theta \\ \frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\sin\theta = \lambda \end{cases}$$

$$\frac{1}{3}ML^2\ddot{\theta} = Mg\frac{L}{2}\sin\theta \Rightarrow \ddot{\theta} = \frac{3g}{2L}\sin\theta$$

$$\dot{\theta}\mathrm{d}\dot{\theta} = \frac{3g}{2L}\sin\theta\mathrm{d}\theta \Rightarrow \int_0^{\dot{\theta}}\dot{\theta}\mathrm{d}\dot{\theta} = \frac{3g}{2L}\int_\alpha^\theta \sin\theta\mathrm{d}\theta \Rightarrow \dot{\theta}^2 = \frac{3g}{L}(\cos\alpha - \cos\theta)$$

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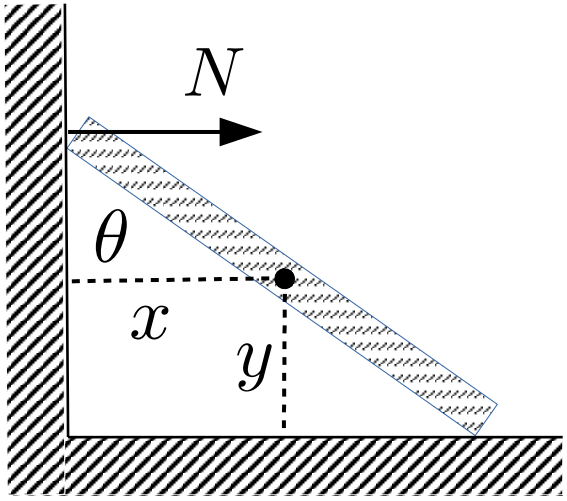
$$\frac{1}{2}ML\ddot{\theta}\cos\theta - \frac{1}{2}ML\dot{\theta}^2\sin\theta = \lambda$$

$$\ddot{\theta} = \frac{3g}{2L}\sin\theta, \quad \dot{\theta}^2 = \frac{3g}{L}(\cos\alpha - \cos\theta)$$

$$\lambda = N = \frac{3}{2}Mg\sin\theta(3\cos\theta - 2\cos\alpha)$$



وقتی نردبان دیوار را تر می‌کند : $N = 0$

$$3\cos\theta - 2\cos\alpha = 0 \Rightarrow 3\cos\theta = 2\cos\alpha$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\cos\alpha\right)$$