جلسه نوزدهم

مکانیک تحلیلی

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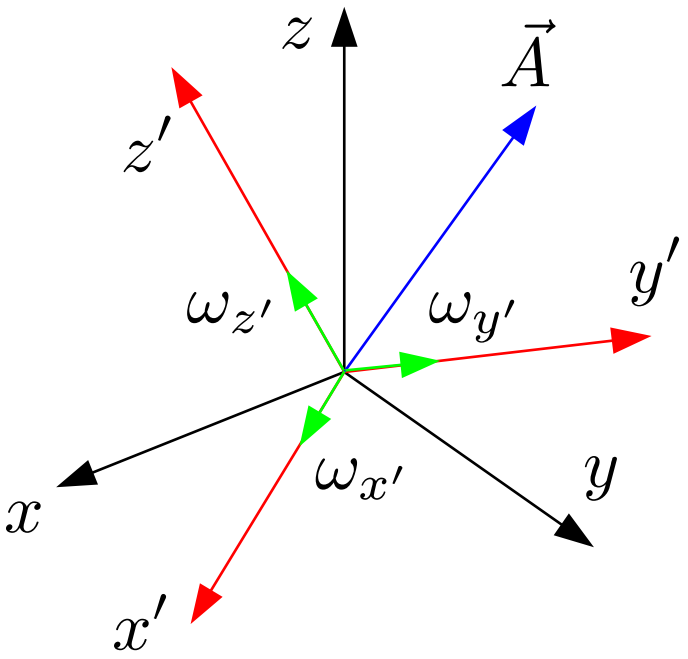
اسفند ۹۸

مکانیک لاگرانژی



مشتق زمانی در دستگاه ثابت و دستگاه چرخان

$$\left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\right)_{\text{ثابت}} = \left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\right)_{\text{چرخان}} + \vec{\omega} \times \vec{r'}$$

$$\vec{A} \to \vec{r}: \quad \vec{v} = \vec{v'} + \vec{\omega} \times \vec{r'}$$

$$T = \frac{1}{2} m \vec{v} \cdot \vec{v} = \frac{1}{2} m (\vec{v'} + \omega \times \vec{r'}) \cdot (\vec{v'} + \omega \times \vec{r'})$$

$$T = \frac{1}{2} m [v'^2 + 2\vec{v'} \cdot \omega \times \vec{r'} + (\omega \times \vec{r'})^2], \qquad V = V(\vec{r'})$$

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مکانیک لاگرانژی

$$T = \frac{1}{2}m[v'^2 + 2\vec{v'} \cdot \omega \times \vec{r'} + (\omega \times \vec{r'})^2], \qquad V = V(\vec{r'})$$

$$\vec{v'} \cdot \omega \times \vec{r'} = \sum_{ijk} \epsilon_{ijk} \dot{x}'_i \omega_j x'_k$$

$$(\omega \times \vec{r'})^2 = (\omega \times \vec{r'}) \cdot (\omega \times \vec{r'}) = \left(\sum_{ijk} \epsilon_{ijk} \hat{e}_i \omega_j x'_k \right) \cdot \left(\sum_{lmn} \epsilon_{lmn} \hat{e}_l \omega_m x'_n \right)$$

$$(\omega \times \vec{r'})^2 = \sum_{ijk} \sum_{lmn} \hat{e}_i \cdot \hat{e}_l \epsilon_{ijk} \epsilon_{lmn} \omega_j x'_k \omega_m x'_n = \sum_{ijk} \sum_{lmn} \delta_{il} \epsilon_{ijk} \epsilon_{lmn} \omega_j x'_k \omega_m x'_n$$

$$(\omega \times \vec{r'})^2 = \sum_{ijk} \sum_{mn} \epsilon_{ijk} \epsilon_{imn} \omega_j x'_k \omega_m x'_n = \sum_{jk} \sum_{mn} \left(\sum_{i} \epsilon_{ijk} \epsilon_{imn} \right) \omega_j x'_k \omega_m x'_n$$

مکانیک لاگرانژی

$$T = \frac{1}{2}m[v'^2 + 2\vec{v'} \cdot \omega \times \vec{r'} + (\omega \times \vec{r'})^2], \qquad V = V(\vec{r'})$$

$$\vec{v'} \cdot \omega \times \vec{r'} = \sum_{ijk} \epsilon_{ijk} \dot{x}'_i \omega_j x'_k$$

$$(\omega \times \vec{r'})^2 = \sum_{jk}\sum_{mn}\left(\sum_{i} \epsilon_{ijk}\epsilon_{imn}\right) \omega_j x'_k \omega_m x'_n$$

$$(\omega \times \vec{r'})^2 = \sum_{jk}\sum_{mn}(\delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km})\omega_j x'_k \omega_m x'_n$$

$$(\omega \times \vec{r'})^2 = \sum_{jk}(\omega_j^2 {x'_k}^2 - \omega_j x'_k \omega_k x'_j)$$

مکانیک لاگرانژی

$$T = \frac{1}{2}m[v'^2 + 2\vec{v'} \cdot \omega \times \vec{r'} + (\omega \times \vec{r'})^2], \qquad V = V(\vec{r'})$$

$$\vec{v'} \cdot \omega \times \vec{r'} = \sum_{ijk} \epsilon_{ijk} \dot{x}'_i \omega_j x'_k$$

$$(\omega \times \vec{r'})^2 = \sum_{jk} (\omega_j^2 {x'_k}^2 - \omega_j x'_k \omega_k x'_j)$$

$$\mathcal{L} = \frac{1}{2}m \sum_i \dot{x}'^2_i + m \sum_{ijk} \epsilon_{ijk} \dot{x}'_i \omega_j x'_k + \frac{1}{2}m \sum_{jk} (\omega_j^2 {x'_k}^2 - \omega_j x'_k \omega_k x'_j) - V(\vec{r'})$$

مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m\sum_i \dot{x}_i'^2 + m\sum_{ijk}\epsilon_{ijk}\dot{x}_i'\omega_j x_k'$$

$$+\frac{1}{2}m\sum_{jk}(\omega_j^2 x_k'^2 - \omega_j x_k'\omega_k x_j') - V(\vec{r'})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}_\alpha'}\right) = \frac{\partial\mathcal{L}}{\partial x_\alpha}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}m\sum_i 2\dot{x}_i'\delta_{i\alpha} + m\sum_{ijk}\epsilon_{ijk}\delta_{i\alpha}\omega_j x_k'\right] = m\sum_{ijk}\epsilon_{ijk}\dot{x}_i'\omega_j\delta_{k\alpha}$$

$$+\frac{1}{2}m\sum_{jk}(\omega_j^2\delta_{k\alpha}2x_k' - \omega_j\delta_{k\alpha}\omega_k x_j' - \omega_j x_k'\omega_k\delta_{j\alpha}) - \frac{\partial V}{\partial x_\alpha'}$$

مکانیک لاگرانژی

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{1}{2}m\sum_i 2\dot{x}'_i\delta_{i\alpha}+m\sum_{ijk}\epsilon_{ijk}\delta_{i\alpha}\omega_j x'_k\right]=m\sum_{ijk}\epsilon_{ijk}\dot{x}'_i\omega_j\delta_{k\alpha}$$

$$+\frac{1}{2}m\sum_{jk}(\omega_j^2\delta_{k\alpha}2x'_k-\omega_j\delta_{k\alpha}\omega_k x'_j-\omega_j x'_k\omega_k\delta_{j\alpha})-\frac{\partial V}{\partial x'_\alpha}$$

$$m\ddot{x}'_\alpha+m\sum_{jk}\epsilon_{\alpha jk}\omega_j\dot{x}'_k=m\sum_{ij}\epsilon_{ij\alpha}\dot{x}'_i\omega_j$$

$$+m(\sum_j\omega_j^2)x'_\alpha-\frac{1}{2}m(\sum_j\omega_j x'_j)\omega_\alpha-\frac{1}{2}m(\sum_k x'_k\omega_k)\omega_\alpha-\frac{\partial V}{\partial x'_\alpha}$$

$$m\ddot{x}'_\alpha=2m\sum_{ij}\epsilon_{ij\alpha}\dot{x}'_i\omega_j+m(\sum_j\omega_j^2)x'_\alpha-m(\sum_k x'_k\omega_k)\omega_\alpha-\frac{\partial V}{\partial x'_\alpha}$$

مکانیک لاگرانژی

$$m\ddot{x}'_\alpha = 2m\sum_{ij}\epsilon_{ij\alpha}\dot{x}'_i\omega_j + m(\sum_j \omega_j^2)x'_\alpha - m(\sum_k x'_k\omega_k)\omega_\alpha - \frac{\partial V}{\partial x'_\alpha}$$

$$\sum_{ij}\epsilon_{ij\alpha}\dot{x}'_i\omega_j = \sum_{ij}\epsilon_{\alpha ij}\dot{x}'_i\omega_j = (\vec{v'}\times\vec{\omega})_\alpha = -(\vec{\omega}\times\vec{v'})_\alpha$$

$$\vec{\omega}\times(\vec{\omega}\times\vec{r'}) = \vec{\omega}(\vec{\omega}\cdot\vec{r'}) - \omega^2\vec{r'}$$

$$[\vec{\omega}\times(\vec{\omega}\times\vec{r'})]_\alpha = \omega_\alpha(\sum_k \omega_k x'_k) - (\sum_k \omega_k^2)x'_\alpha$$

$$m\ddot{x}'_\alpha = -2m(\vec{\omega}\times\vec{v'})_\alpha - m[\vec{\omega}\times(\vec{\omega}\times\vec{r'})]_\alpha - \frac{\partial V}{\partial x'_\alpha}$$

مکانیک لاگرانژی

$$m\ddot{x}'_\alpha = -2m(\vec{\omega} \times \vec{v'})_\alpha - m[\vec{\omega} \times (\vec{\omega} \times \vec{r'})]_\alpha - \frac{\partial V}{\partial x'_\alpha}$$

$$F_\alpha = -\frac{\partial V}{\partial x'_\alpha}$$

$$m\ddot{x}'_\alpha = -2m(\vec{\omega} \times \vec{v'})_\alpha - m[\vec{\omega} \times (\vec{\omega} \times \vec{r'})]_\alpha + F_\alpha$$

$$m\ddot{\vec{r'}} = -2m(\vec{\omega} \times \vec{v'}) - m\vec{\omega} \times (\vec{\omega} \times \vec{r'}) + \vec{F}$$

مکانیک لاگرانژی

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{v} = \hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z} \qquad \vec{B} = \hat{i}B_x + \hat{j}B_y + \hat{k}B_z$$

$$\vec{F} = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

$$m\ddot{x}\hat{i} + m\ddot{y}\hat{j} + \ddot{z}\hat{k} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dot{x} & \dot{y} & \dot{z} \\ B_x & B_y & B_z \end{vmatrix}$$

معادلات حرکت

$$m\ddot{x} = q(\dot{y}B_z - \dot{z}B_y), \quad m\ddot{y} = q(\dot{z}B_x - \dot{x}B_z), \quad m\ddot{z} = q(\dot{x}B_y - \dot{y}B_x)$$

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مکانیک لاگرانژی

لاگرانژ ذره باردار q در میدان مغناطیس $\vec{B}$

$$\mathcal{L} = \frac{1}{2}m\vec{v}\cdot\vec{v} + q\vec{v}\cdot\vec{A}$$

$$\vec{v} = \hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}$$

پتانسیل برداری : $\vec{A}(x,y,z,t) = \hat{i}A_x(x,y,z,t) + \hat{j}A_y(x,y,z,t) + \hat{k}A_z(x,y,z,t)$

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}, \quad \nabla = \hat{i}\partial_x + \hat{j}\partial_y + \hat{k}\partial_z$$

هدف بدست آوردن معادلات حرکت

$$m\ddot{x} = q(\dot{y}B_z - \dot{z}B_y), \quad m\ddot{y} = q(\dot{z}B_x - \dot{x}B_z), \quad m\ddot{z} = q(\dot{x}B_y - \dot{y}B_x)$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2} m \vec{v} \cdot \vec{v} + q \vec{v} \cdot \vec{A}$$

$$\vec{v} = \hat{i}\dot{x} + \hat{j}\dot{y} + \hat{k}\dot{z}, \quad \vec{v} \cdot \vec{v} = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

$$\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z, \quad \vec{v} \cdot \vec{A} = \dot{x}A_x + \dot{y}A_y + \dot{z}A_z$$

$$\mathcal{L} = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$\vec{A}(x, y, z, t) = \hat{i}A_x(x, y, z, t) + \hat{j}A_y(x, y, z, t) + \hat{k}A_z(x, y, z, t)$$

فرض می‌کنیم که

$$\vec{A}(x, y, z) = \hat{i}A_x(x, y, z) + \hat{j}A_y(x, y, z) + \hat{k}A_z(x, y, z)$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$\vec{A}(x,y,z) = \hat{i}A_x(x,y,z) + \hat{j}A_y(x,y,z) + \hat{k}A_z(x,y,z)$$

مولفه‌ی x

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m\dot{x} + qA_x\right] = q\dot{x}\left(\frac{\partial A_x}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x}\right)$$

$$m\ddot{x} + q\frac{\mathrm{d}A_x}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x}\right)$$

$$\frac{\mathrm{d}A_x}{\mathrm{d}t} = ?$$

مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$\vec{A}(x,y,z) = \hat{i}A_x(x,y,z) + \hat{j}A_y(x,y,z) + \hat{k}A_z(x,y,z)$$

مولفه‌ی y

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) = \frac{\partial \mathcal{L}}{\partial y}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m\dot{y} + qA_y\right] = q\dot{x}\left(\frac{\partial A_x}{\partial y}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y}\right)$$

$$m\ddot{y} + q\frac{\mathrm{d}A_y}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial y}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y}\right)$$

$$\frac{\mathrm{d}A_y}{\mathrm{d}t} = ?$$

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$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$\vec{A}(x,y,z) = \hat{i}A_x(x,y,z) + \hat{j}A_y(x,y,z) + \hat{k}A_z(x,y,z)$$

مولفه‌ی z

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{z}}\right) = \frac{\partial \mathcal{L}}{\partial z}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[m\dot{z} + qA_z\right] = q\dot{x}\left(\frac{\partial A_x}{\partial z}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial z}\right)$$

$$m\ddot{z} + q\frac{\mathrm{d}A_z}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial z}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial z}\right)$$

$$\frac{\mathrm{d}A_z}{\mathrm{d}t} = ?$$

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$$A_x = A_x(x,y,z) \Rightarrow \mathrm{d}A_x = \left(\frac{\partial A_x}{\partial x}\right)\mathrm{d}x + \left(\frac{\partial A_x}{\partial y}\right)\mathrm{d}y + \left(\frac{\partial A_x}{\partial z}\right)\mathrm{d}z$$

$$\boxed{\frac{\mathrm{d}A_x}{\mathrm{d}t} = \left(\frac{\partial A_x}{\partial x}\right)\dot{x} + \left(\frac{\partial A_x}{\partial y}\right)\dot{y} + \left(\frac{\partial A_x}{\partial z}\right)\dot{z}}$$

$$A_y = A_y(x,y,z) \Rightarrow \mathrm{d}A_y = \left(\frac{\partial A_y}{\partial x}\right)\mathrm{d}x + \left(\frac{\partial A_y}{\partial y}\right)\mathrm{d}y + \left(\frac{\partial A_y}{\partial z}\right)\mathrm{d}z$$

$$\boxed{\frac{\mathrm{d}A_y}{\mathrm{d}t} = \left(\frac{\partial A_y}{\partial x}\right)\dot{x} + \left(\frac{\partial A_y}{\partial y}\right)\dot{y} + \left(\frac{\partial A_y}{\partial z}\right)\dot{z}}$$

$$A_z = A_z(x,y,z) \Rightarrow \mathrm{d}A_z = \left(\frac{\partial A_z}{\partial x}\right)\mathrm{d}x + \left(\frac{\partial A_z}{\partial y}\right)\mathrm{d}y + \left(\frac{\partial A_z}{\partial z}\right)\mathrm{d}z$$

$$\boxed{\frac{\mathrm{d}A_z}{\mathrm{d}t} = \left(\frac{\partial A_z}{\partial x}\right)\dot{x} + \left(\frac{\partial A_z}{\partial y}\right)\dot{y} + \left(\frac{\partial A_z}{\partial z}\right)\dot{z}}$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$m\ddot{x} + q\frac{\mathrm{d}A_x}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x}\right)$$

مولفه‌ی x

$$\frac{\mathrm{d}A_x}{\mathrm{d}t} = \left(\frac{\partial A_x}{\partial x}\right)\dot{x} + \left(\frac{\partial A_x}{\partial y}\right)\dot{y} + \left(\frac{\partial A_x}{\partial z}\right)\dot{z}$$

$$m\ddot{x} + q\left[\left(\frac{\partial A_x}{\partial x}\right)\dot{x} + \left(\frac{\partial A_x}{\partial y}\right)\dot{y} + \left(\frac{\partial A_x}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x}\right)$$

مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

مولفه‌ی x

$$m\ddot{x} + q\left[\left(\frac{\partial A_x}{\partial x}\right)\dot{x} + \left(\frac{\partial A_x}{\partial y}\right)\dot{y} + \left(\frac{\partial A_x}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x}\right)$$

$$m\ddot{x} = q\dot{y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

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مولفه‌ی x

$$m\ddot{x} = q\dot{y}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \hat{i}\underbrace{\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)}_{B_x} + \hat{j}\underbrace{\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)}_{B_y} + \hat{k}\underbrace{\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)}_{B_z}$$

$$m\ddot{x} = q\dot{y}B_z - q\dot{z}B_y$$

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$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$m\ddot{y} + q\frac{\mathrm{d}A_y}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial y}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y}\right)$$

مولفه‌ی y

$$\frac{\mathrm{d}A_y}{\mathrm{d}t} = \left(\frac{\partial A_y}{\partial x}\right)\dot{x} + \left(\frac{\partial A_y}{\partial y}\right)\dot{y} + \left(\frac{\partial A_y}{\partial z}\right)\dot{z}$$

$$m\ddot{y} + q\left[\left(\frac{\partial A_y}{\partial x}\right)\dot{x} + \left(\frac{\partial A_y}{\partial y}\right)\dot{y} + \left(\frac{\partial A_y}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial y}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y}\right)$$

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مولفه‌ی y

$$m\ddot{y} + q\left[\left(\frac{\partial A_y}{\partial x}\right)\dot{x} + \left(\frac{\partial A_y}{\partial y}\right)\dot{y} + \left(\frac{\partial A_y}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial y}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial y}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y}\right)$$

$$m\ddot{y} = q\dot{x}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$$

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$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

مولفه‌ی y

$$m\ddot{y} = q\dot{x}\left(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{B} = \hat{i}\underbrace{\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)}_{B_x} + \hat{j}\underbrace{\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)}_{B_y} + \hat{k}\underbrace{\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)}_{B_z}$$

$$m\ddot{y} = -q\dot{x}B_z + q\dot{z}B_x$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

$$m\ddot{z} + q\frac{\mathrm{d}A_z}{\mathrm{d}t} = q\dot{x}\left(\frac{\partial A_x}{\partial z}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial z}\right)$$

مولفه‌ی z

$$\frac{\mathrm{d}A_z}{\mathrm{d}t} = \left(\frac{\partial A_z}{\partial x}\right)\dot{x} + \left(\frac{\partial A_z}{\partial y}\right)\dot{y} + \left(\frac{\partial A_z}{\partial z}\right)\dot{z}$$

$$m\ddot{z} + q\left[\left(\frac{\partial A_z}{\partial x}\right)\dot{x} + \left(\frac{\partial A_z}{\partial y}\right)\dot{y} + \left(\frac{\partial A_z}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial z}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial z}\right)$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

مولفه‌ی z

$$m\ddot{z} + q\left[\left(\frac{\partial A_z}{\partial x}\right)\dot{x} + \left(\frac{\partial A_z}{\partial y}\right)\dot{y} + \left(\frac{\partial A_z}{\partial z}\right)\dot{z}\right]$$

$$= q\dot{x}\left(\frac{\partial A_x}{\partial z}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z}\right) + q\dot{z}\left(\frac{\partial A_z}{\partial z}\right)$$

$$m\ddot{z} = q\dot{x}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}\right)$$

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مکانیک لاگرانژی

$$\mathcal{L} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + q(\dot{x}A_x + \dot{y}A_y + \dot{z}A_z)$$

مولفه‌ی z

$$m\ddot{z} = q\dot{x}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + q\dot{y}\left(\frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}\right)$$

$$\vec{B} = \nabla \times \vec{A}$$

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$$m\ddot{z} = q\dot{x}B_y - q\dot{y}B_x$$

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