Computational Physics

Lecture-11

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Contents

Numerical Finding Root

- **Secant**

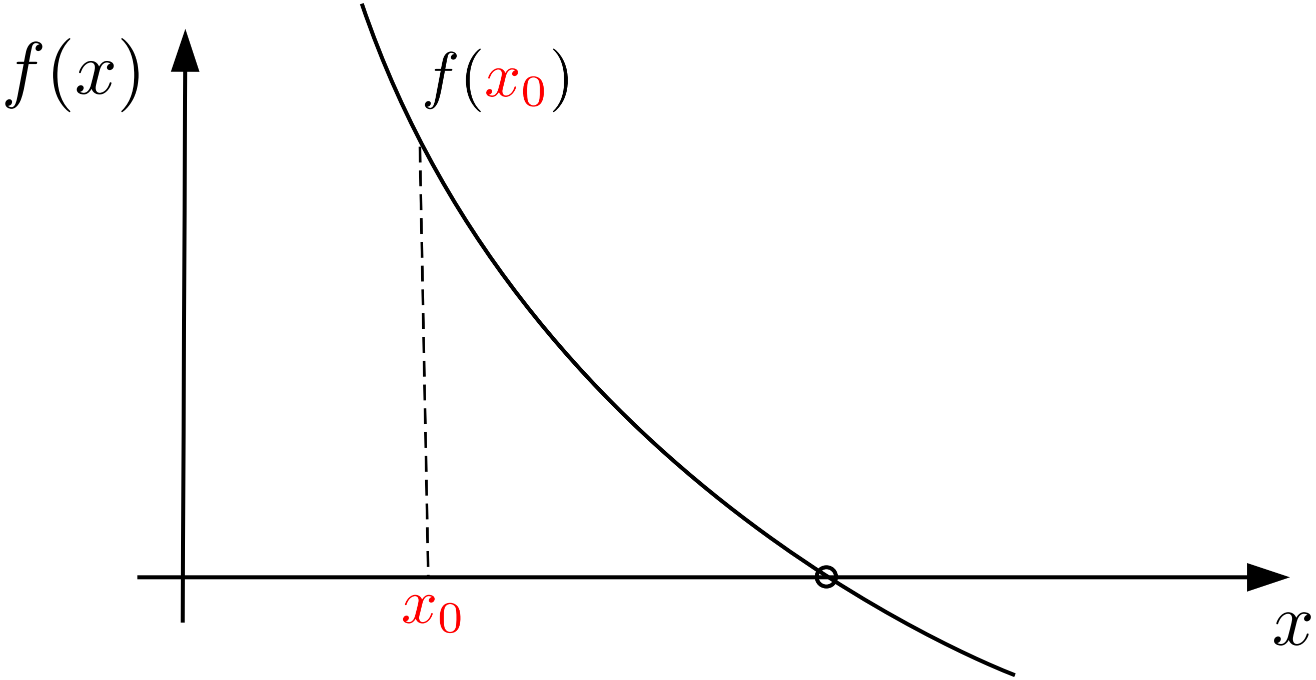
Step 0: An initial guess x_0

And interval spacing h

or

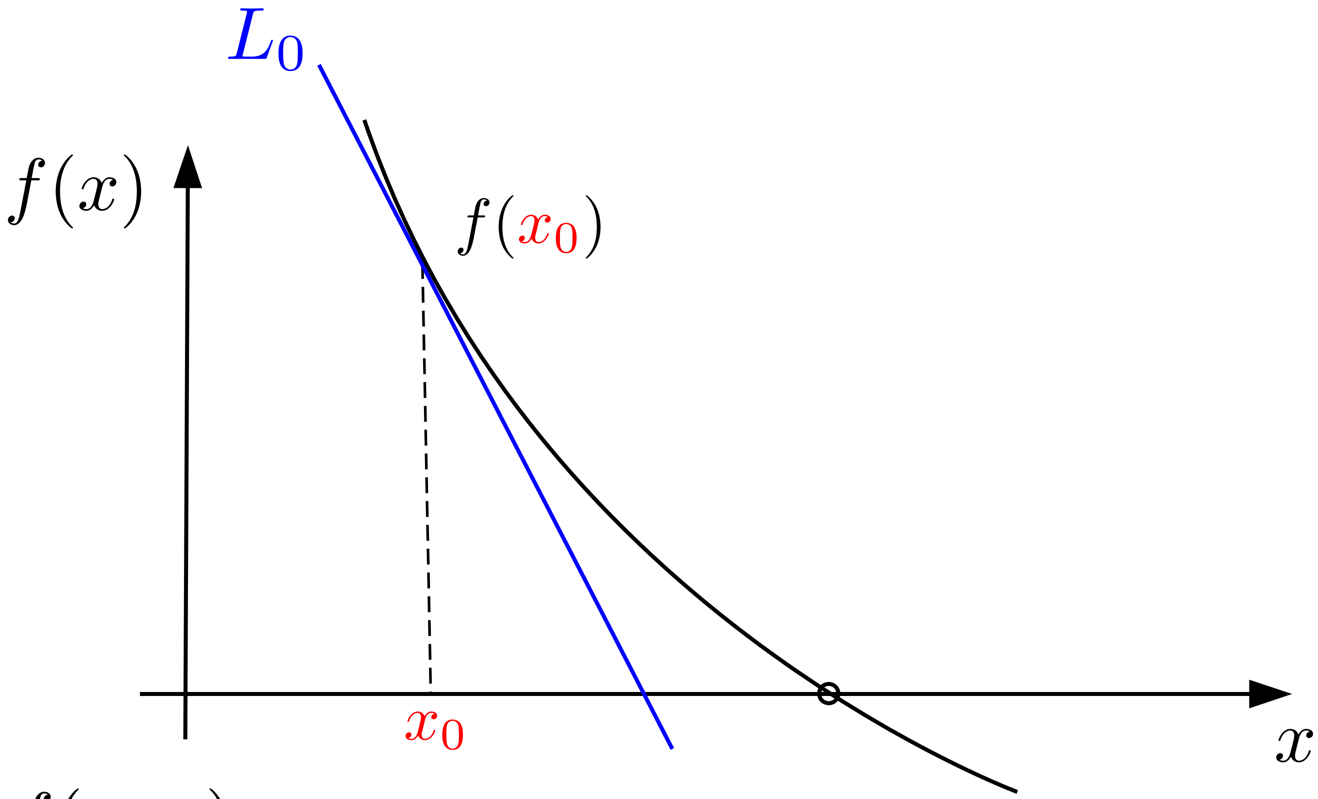
Step 0: An initial guess x_{-1}

and x_0



Numerical Finding Root

- **Secant**



Step 1: The equation of the tangent line,

$$L_0 : \ y = f(x_0) + m(x - x_0)$$

$$m = \frac{f(x_0) - f(x_0 - h)}{h} = \frac{f(x_0) - f(x_{-1})}{x_0 - x_{-1}}$$

Numerical Finding Root

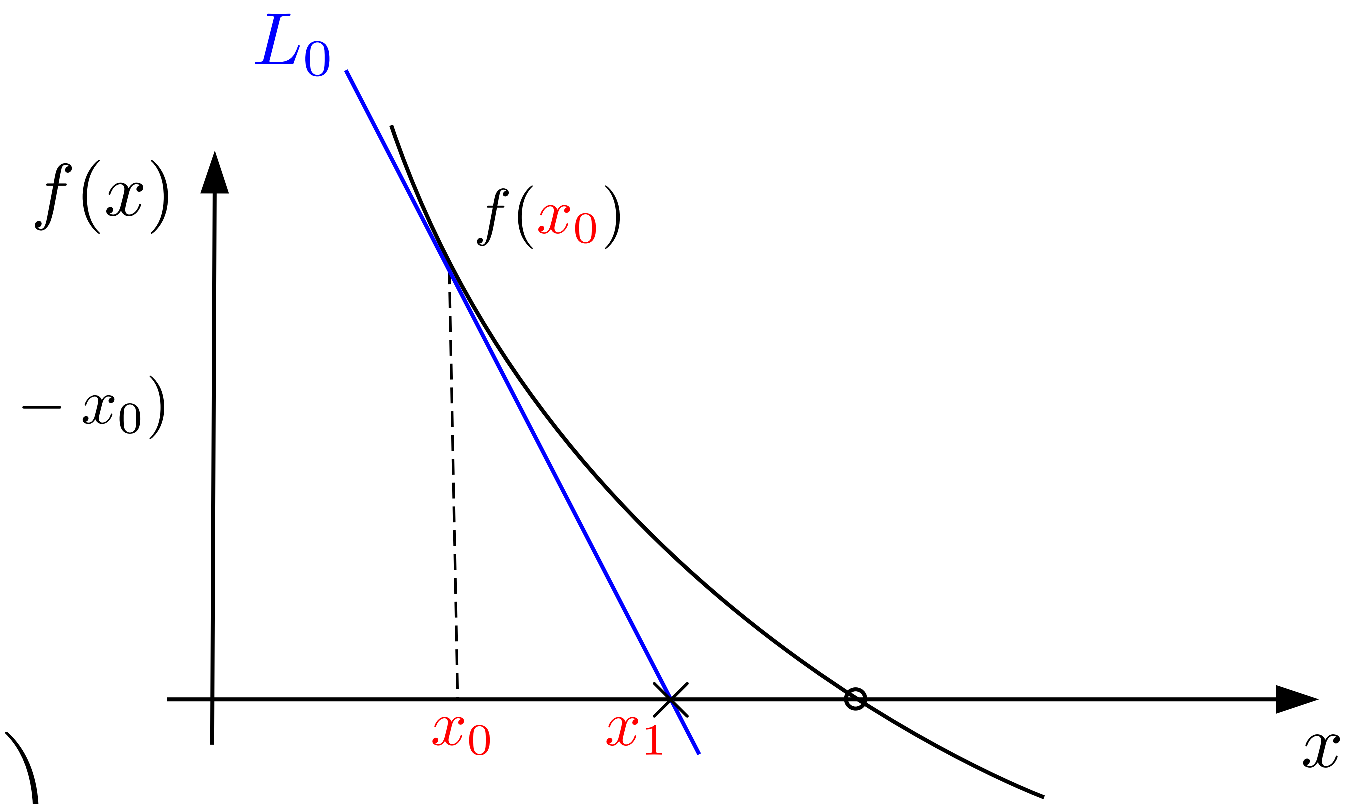
- **Secant**

Step 2: The tangent line intersects the x-axis and new point,

$$L_0 : y = f(x_0) + \left(\frac{f(x_0) - f(x_{-1})}{x_0 - x_{-1}}\right)(x - x_0)$$

$$y(x = x_1) = 0$$

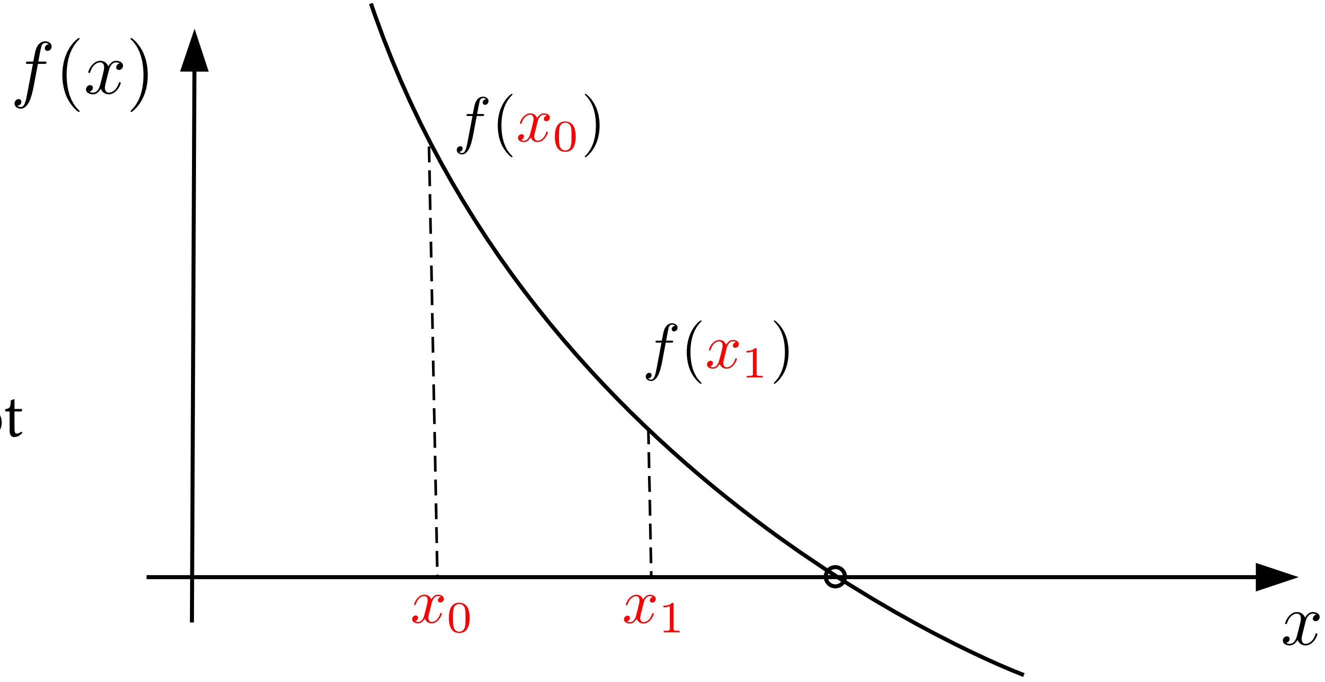
$$x_1 = x_0 - f(x_0)\left(\frac{x_0 - x_{-1}}{f(x_0) - f(x_{-1})}\right)$$



Numerical Finding Root

- **Secant**

$$x_1 = x_0 - f(x_0)\left(\frac{x_0 - x_{-1}}{f(x_0) - f(x_{-1})}\right)$$



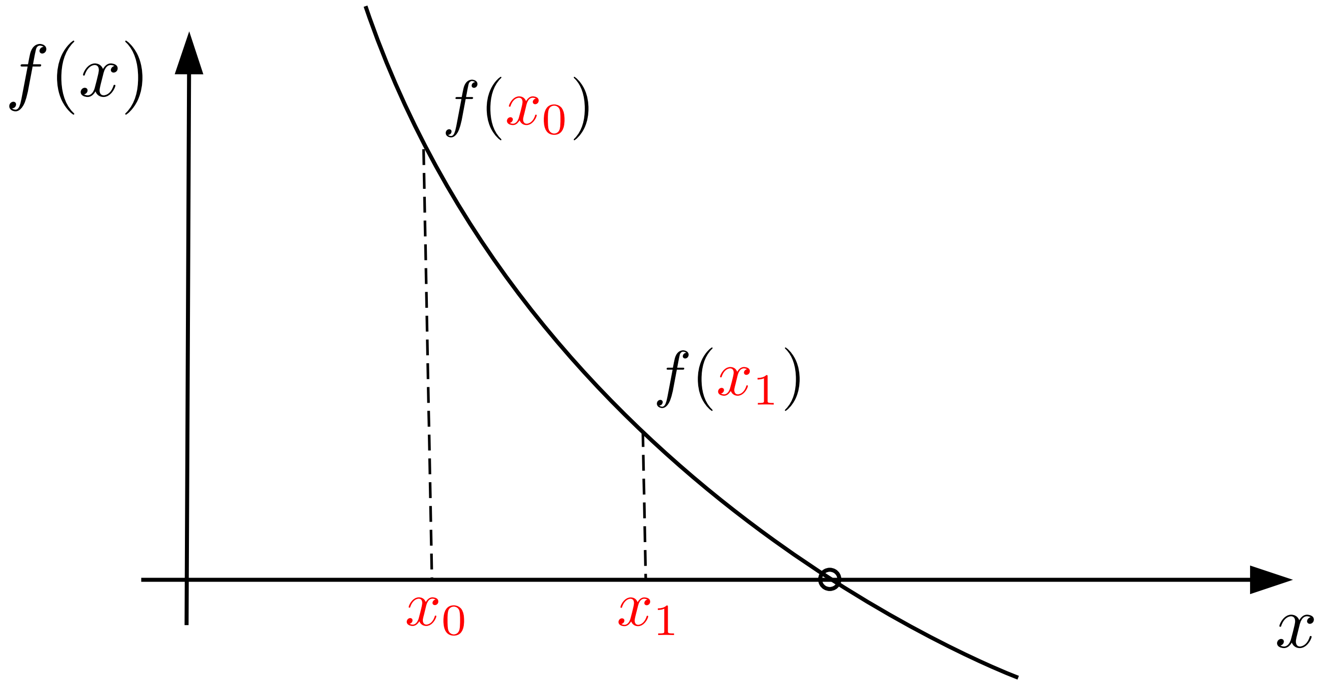
Step 3: Check if it has **converged** to a root within acceptable precision

$$|x_1 - x_0| < \epsilon, \quad \epsilon \ll 1$$

Numerical Finding Root

- **Secant**

Step 4: Return to **Step 1**.



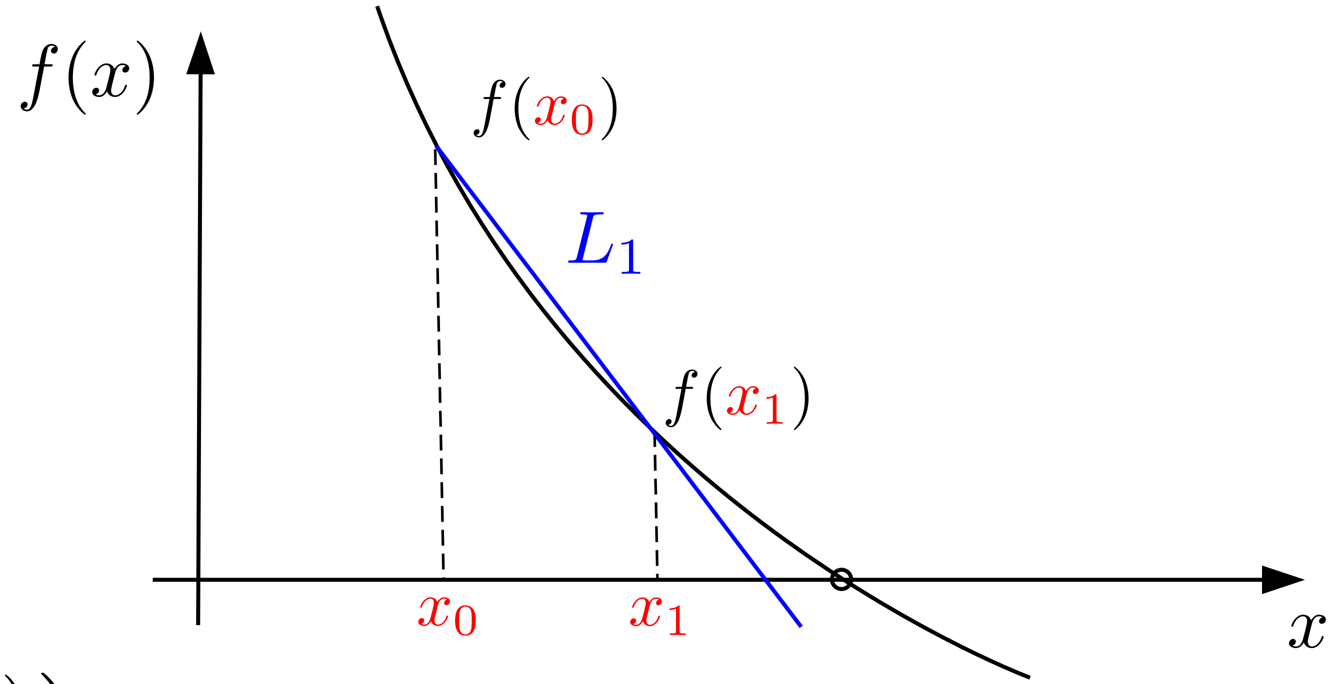
Numerical Finding Root

- **Secant**

Step 1: The equation of the tangent line,

$$L_1 : y = f(x_1) + m(x - x_1)$$

$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



$$L_1 : y = f(x_1) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_1)$$

Numerical Finding Root

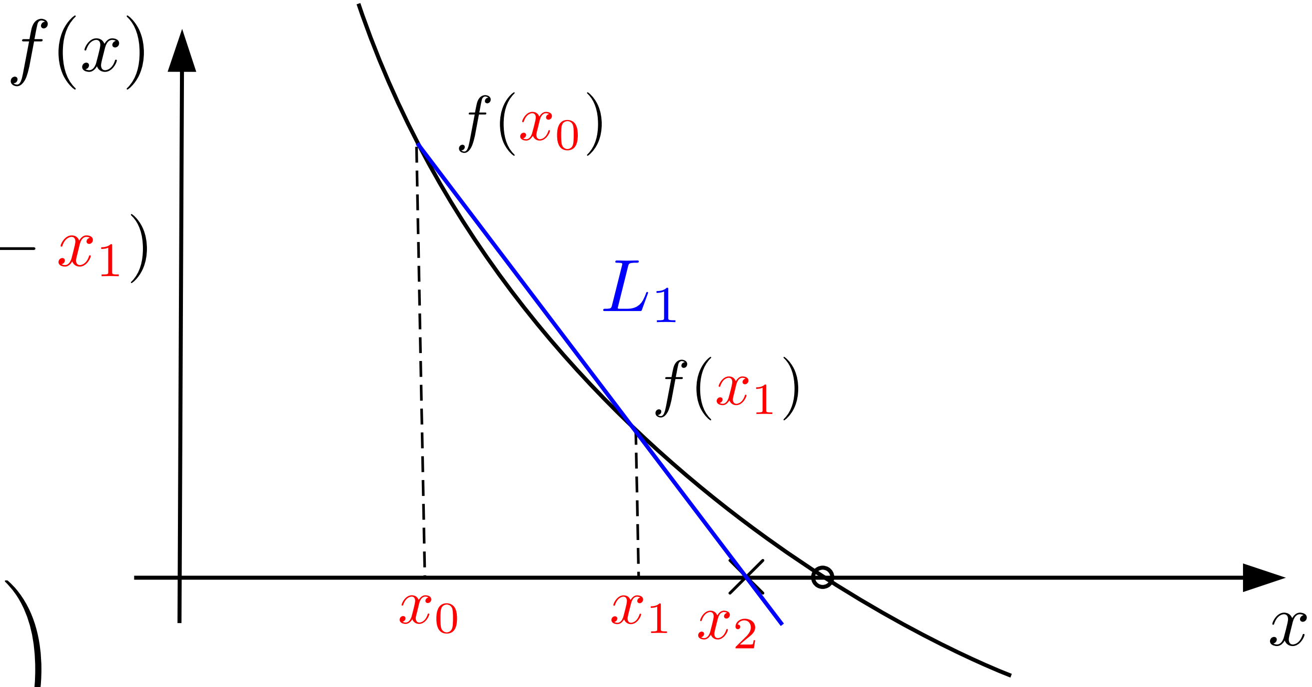
- **Secant**

Step 2: The tangent line intersects the x-axis and new point,

$$L_1 \,:\, y = f(x_1) + \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)(x - x_1)$$

$$y(x_2) = 0$$

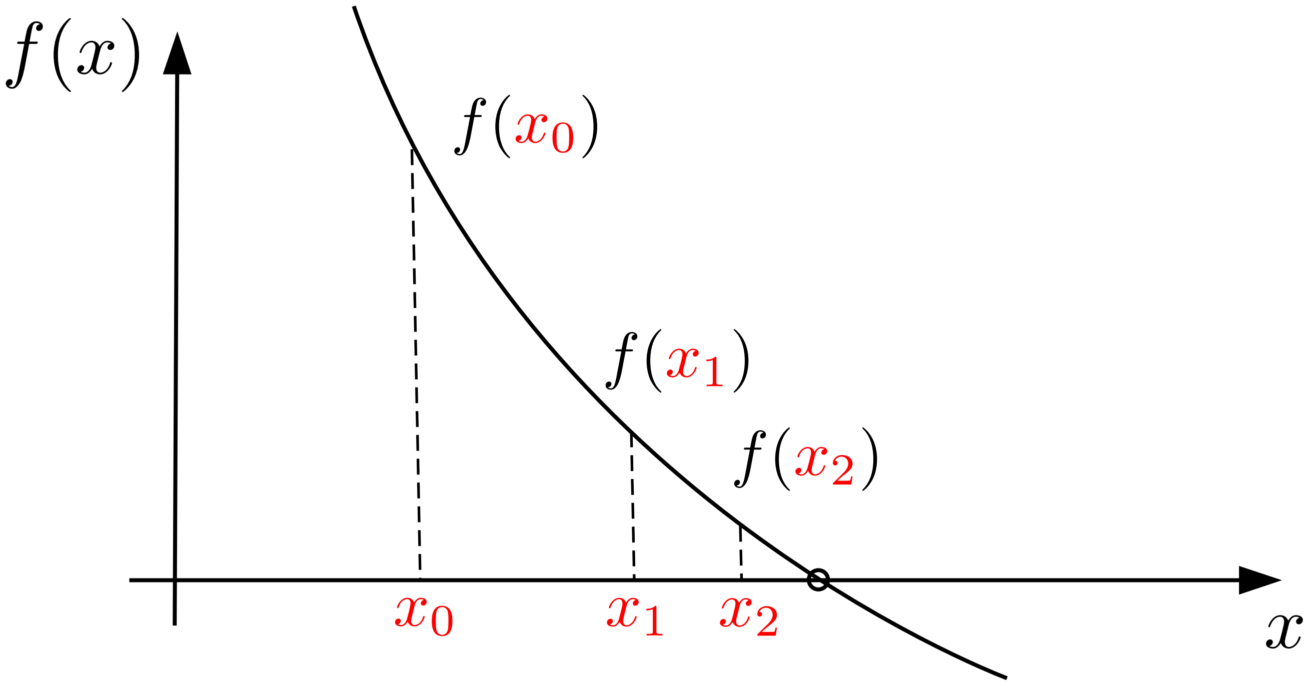
$$x_2 = x_1 - f(x_1)\left(\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right)$$



Numerical Finding Root

- **Secant**

$$x_2 = x_1 - f(x_1)\left(\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right)$$



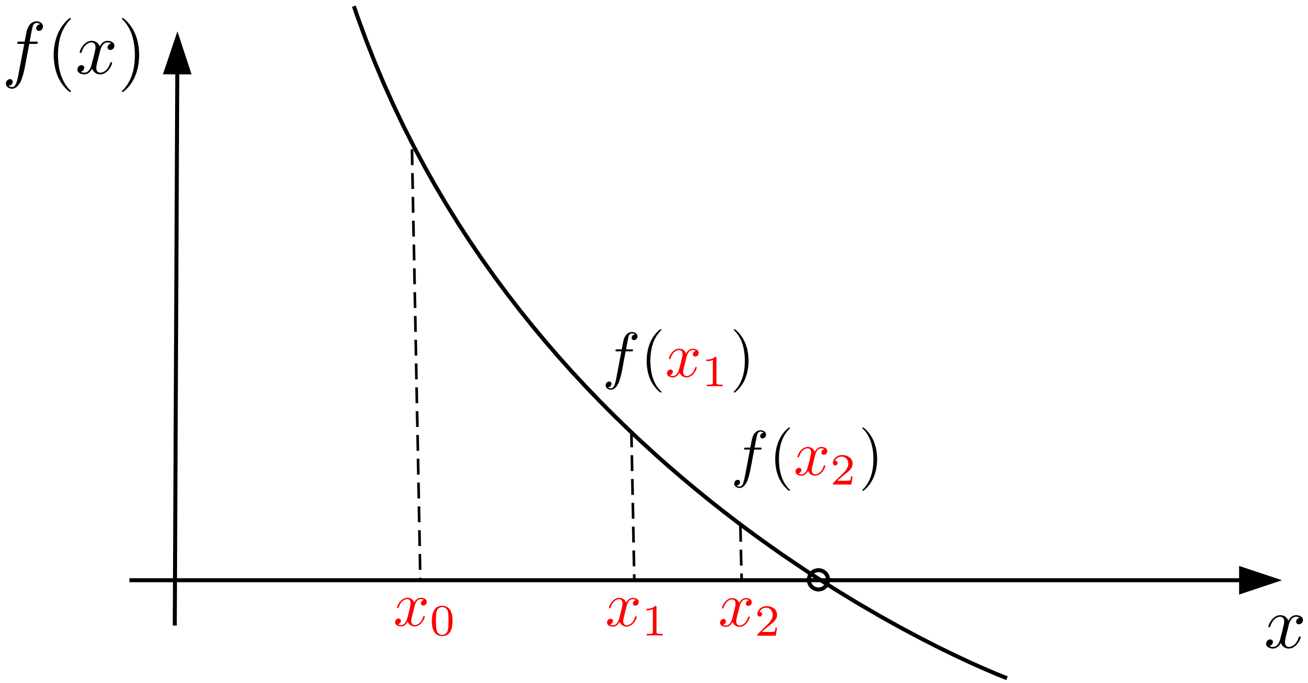
Step 3: Check if it has **converged** to a root within acceptable precision

$$|x_2 - x_1| < \epsilon, \quad \epsilon \ll 1$$

Numerical Finding Root

- **Secant**

Step 4: Return to **Step 1**.



Numerical Finding Root

- **Secant**

Step 1: The equation of the tangent line,

$$L_n:\ y = f(x_n) + m(x - x_n)$$

$$m = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Step 2: The tangent line intersects the x-axis and new point,

$$y(x_{n+1}) = 0$$

$$x_{n+1} = x_n - f(x_n)\left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right)$$

Step 3: Check if it has **converged** to a root within acceptable precision

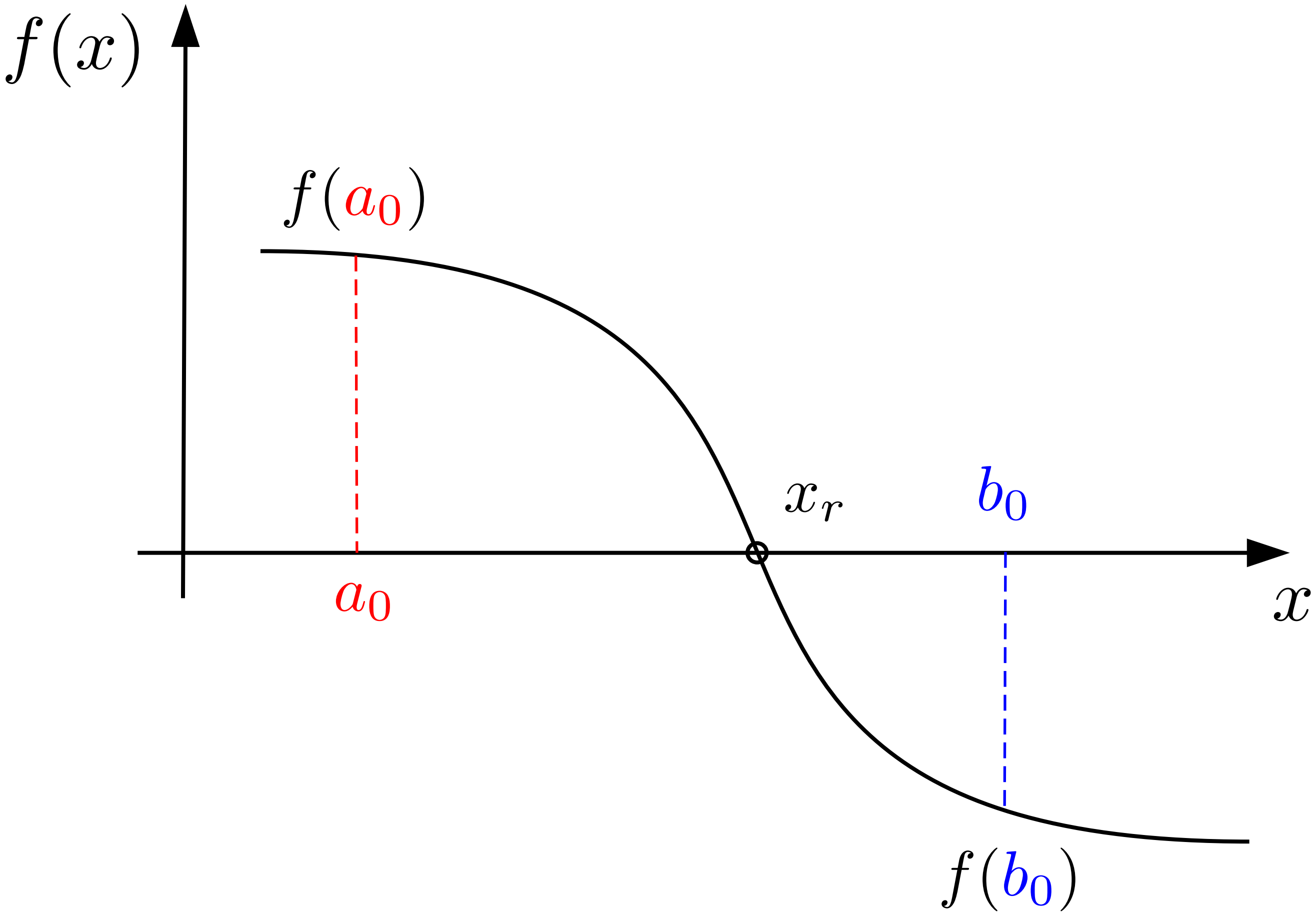
$$|x_{n+1} - x_n| < \epsilon, \quad \epsilon \ll 1$$

Step 4: Return to **Step 1**

Numerical Finding Root

- **False Position Method (Secant Method + Bracketed Method)**

It turns out that it is quite easy to rewrite the **Secant method** as a **Bracketed method**, to significantly **increase the rate of convergence**-the result is the method of **False Positions**.



Step 0: Choose an appropriate interval $x \in [a_0, b_0]$, such that $f(a_0)f(b_0) < 0$

Numerical Finding Root

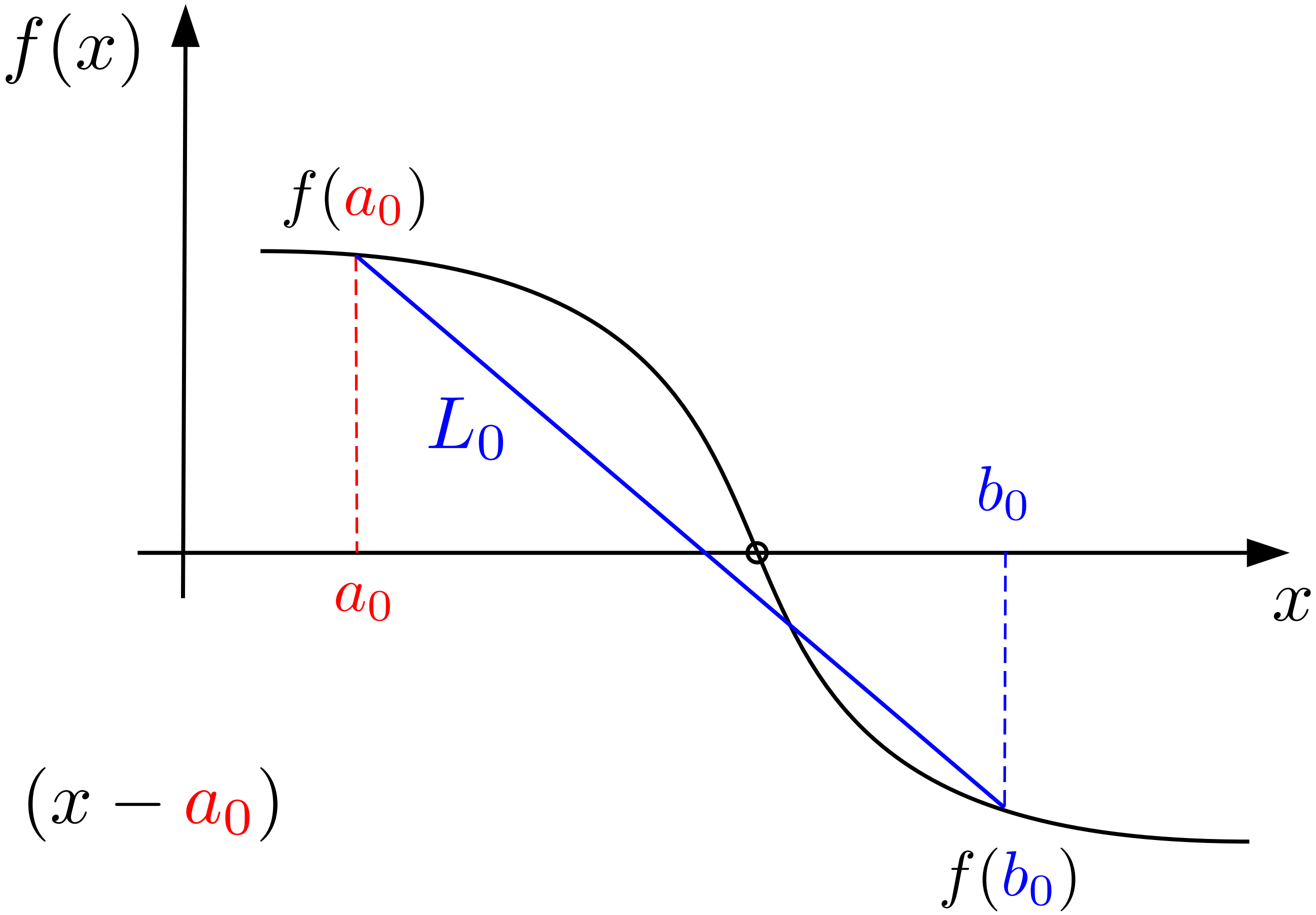
- **False Position Method**

Step 1: The equation of the line,

$$L_0 : \; y = f(a_0) + m(x - a_0)$$

$$m = \frac{f(b_0) - f(a_0)}{b_0 - a_0}$$

$$L_0 : \; y = f(a_0) + \left(\frac{f(b_0) - f(a_0)}{b_0 - a_0}\right)(x - a_0)$$



Numerical Finding Root

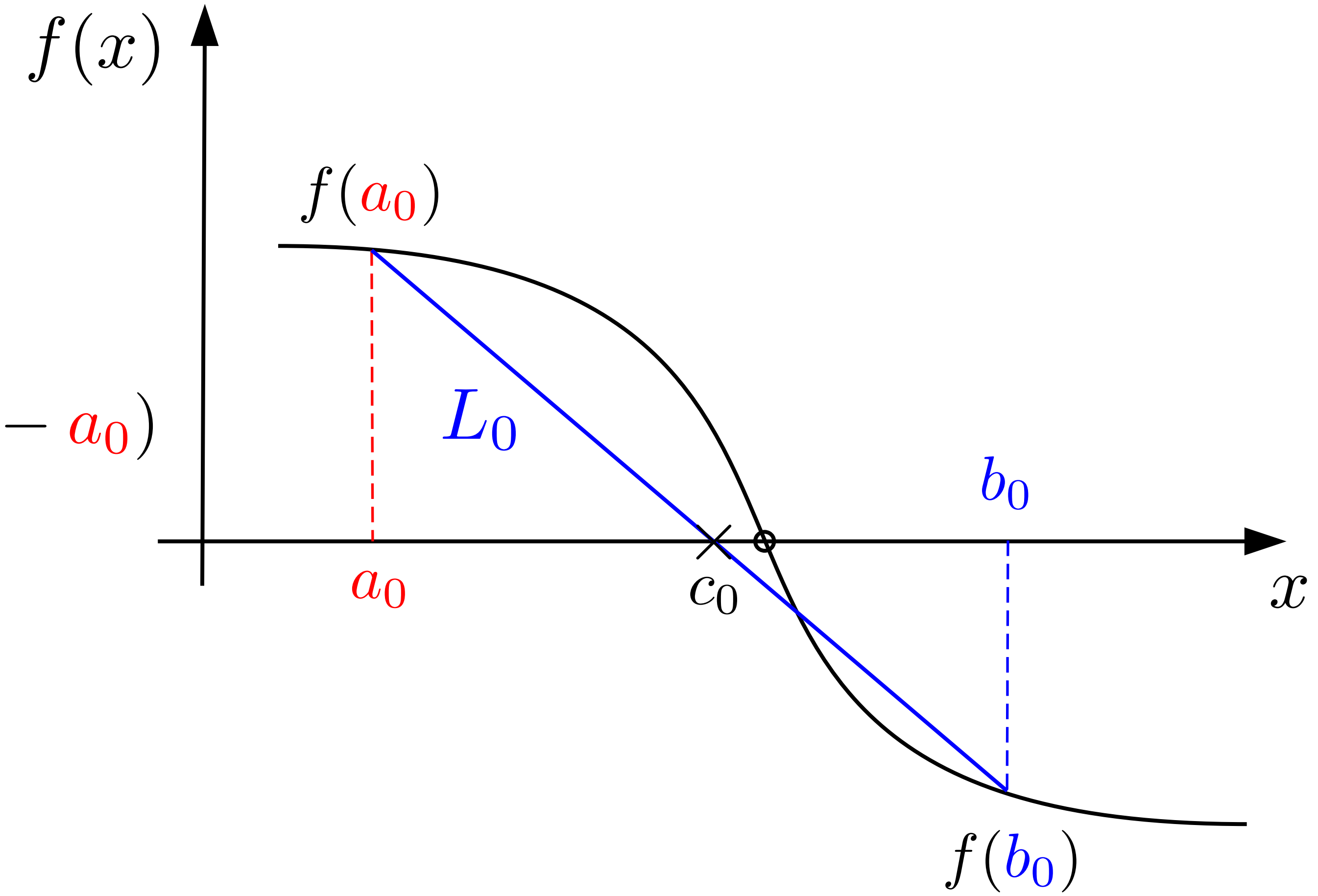
- **False Position Method**

Step 1: The line intersects the x-axis and new point,

$$L_0 : y = f(a_0) + \left(\frac{f(b_0) - f(a_0)}{b_0 - a_0}\right)(x - a_0)$$

$$y(c_0) = 0$$

$$c_0 = a_0 - f(a_0)\left(\frac{b_0 - a_0}{f(b_0) - f(a_0)}\right)$$

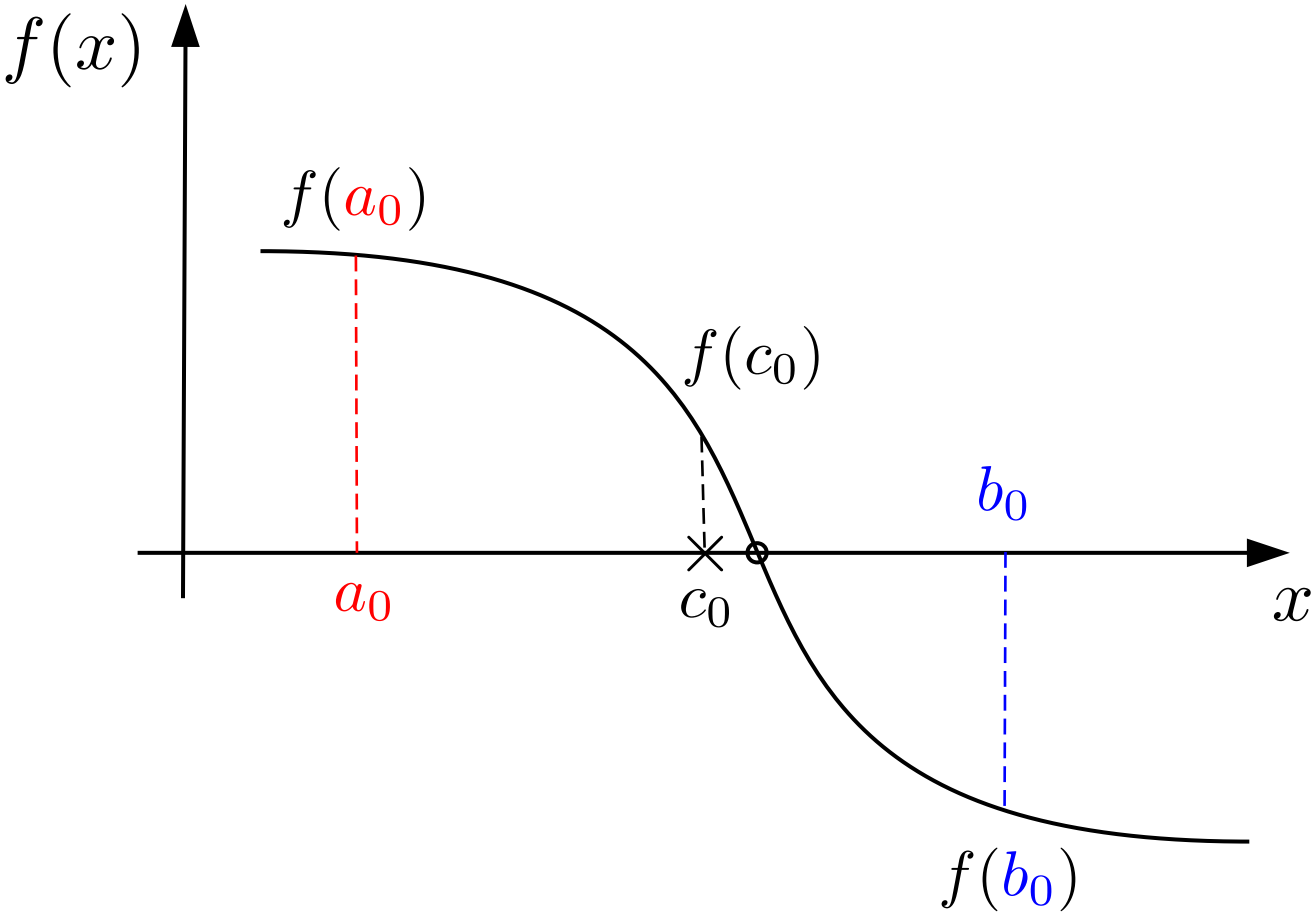


Numerical Finding Root

- **False Position Method**

Step 2: Check if it has **converged** to a root within acceptable precision,

$$|f(c_0)| < \epsilon, \quad \epsilon \ll 1$$



Numerical Finding Root

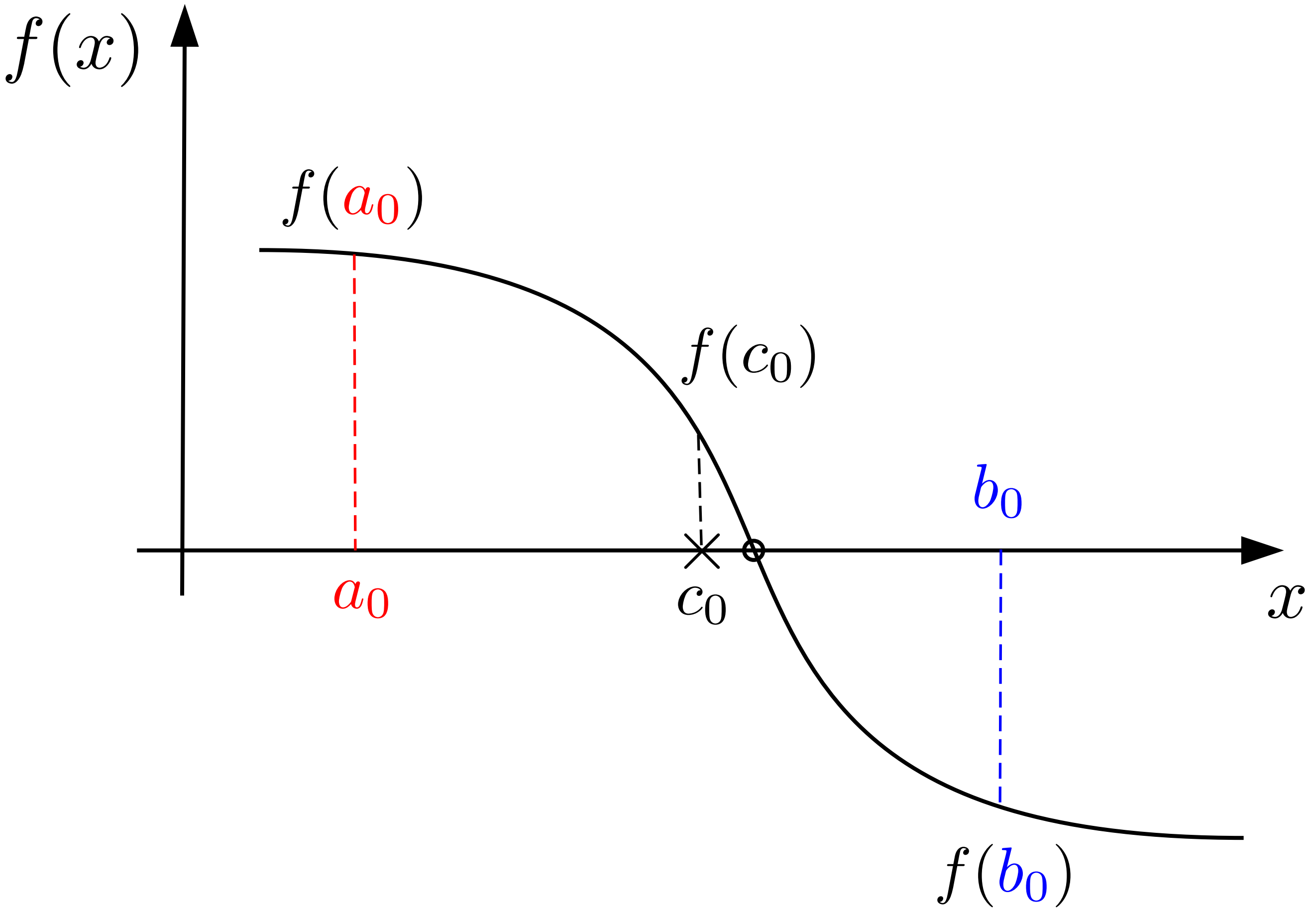
- **False Position Method**

Step 3:

New interval,

$$f(a_0)f(c_0) > 0$$

$$f(a_0)f(c_0) < 0$$



Numerical Finding Root

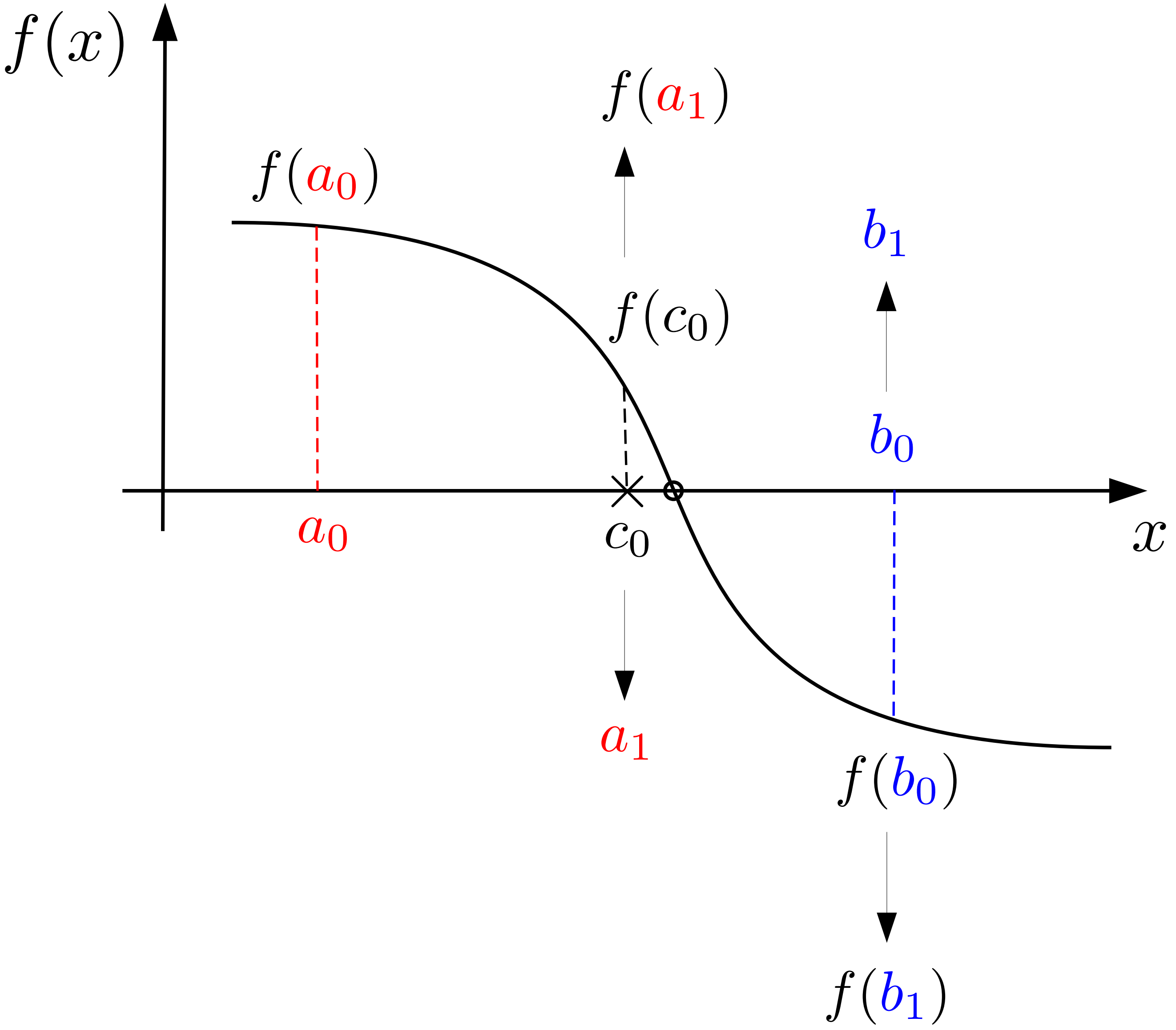
- **False Position Method**

Step 3:

New interval,

$$f(a_0)f(c_0) > 0$$
$$[a_1, b_1] = [c_0, b_0]$$

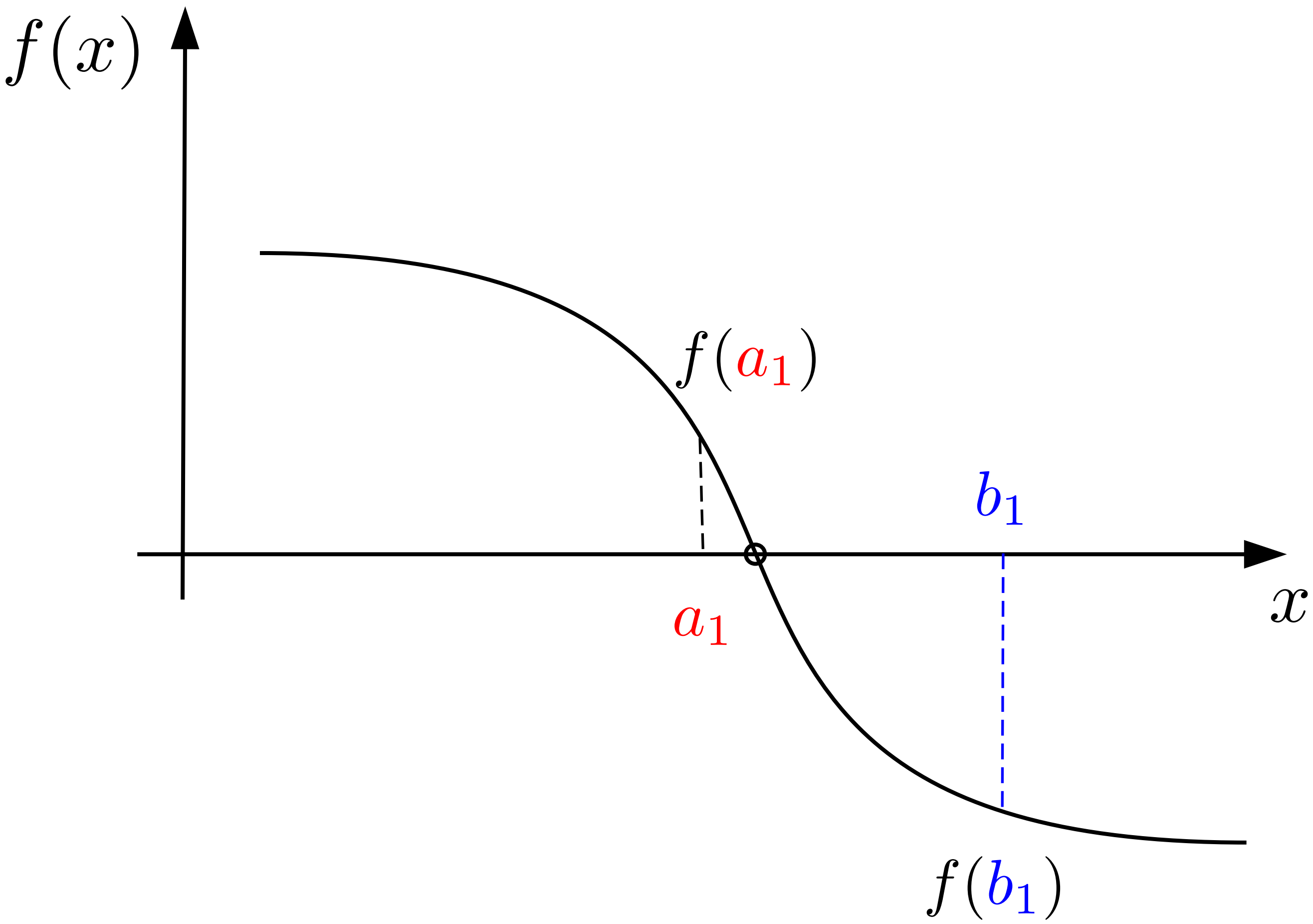
$$f(a_0)f(c_0) < 0$$
$$[a_1, b_1] = [a_0, c_0]$$



Numerical Finding Root

- **False Position Method**

Step 4: Return **step 1**



Numerical Finding Root

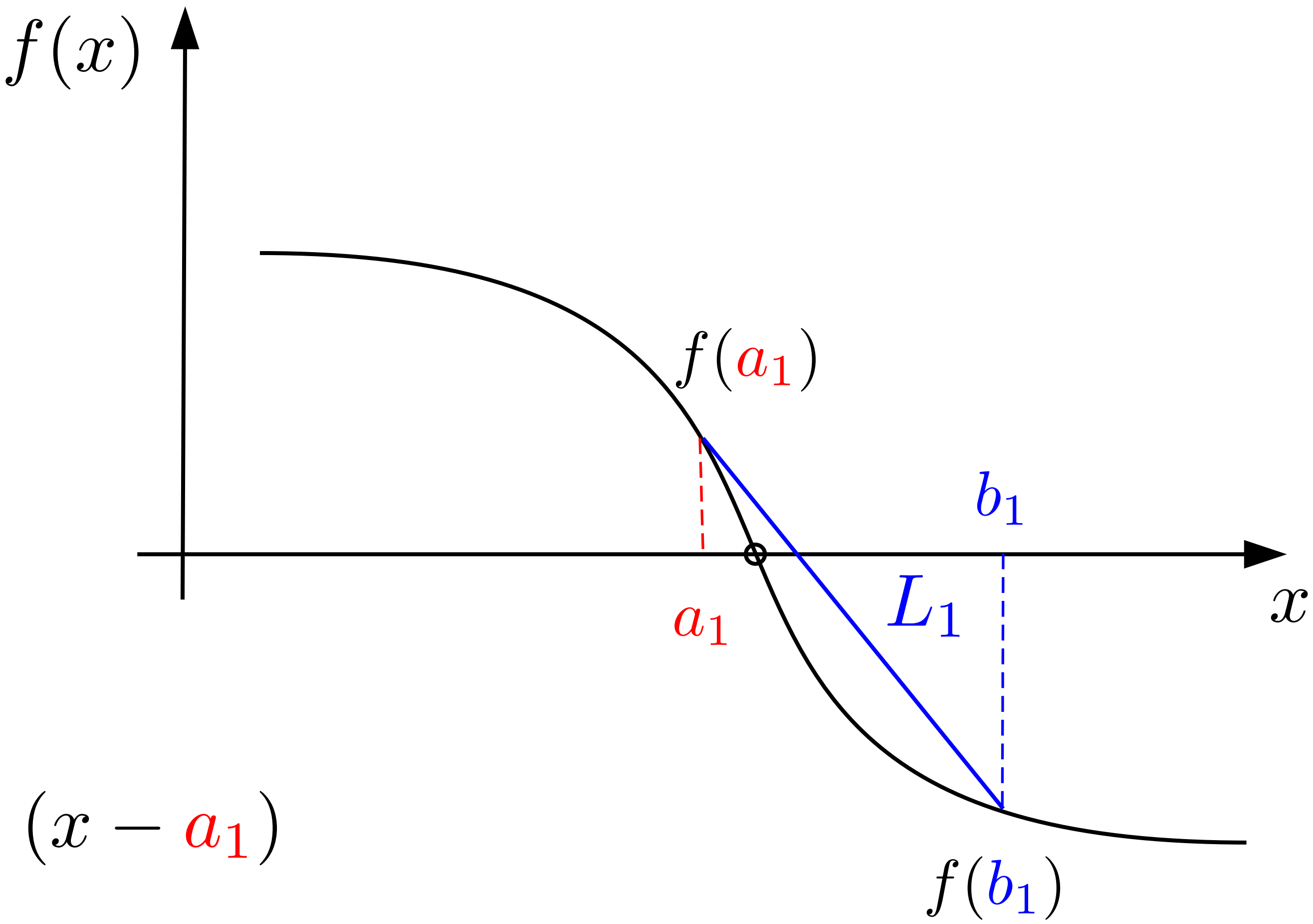
- **False Position Method**

Step 1: The equation of the line,

$$L_1 : y = f(a_1) + m(x - a_1)$$

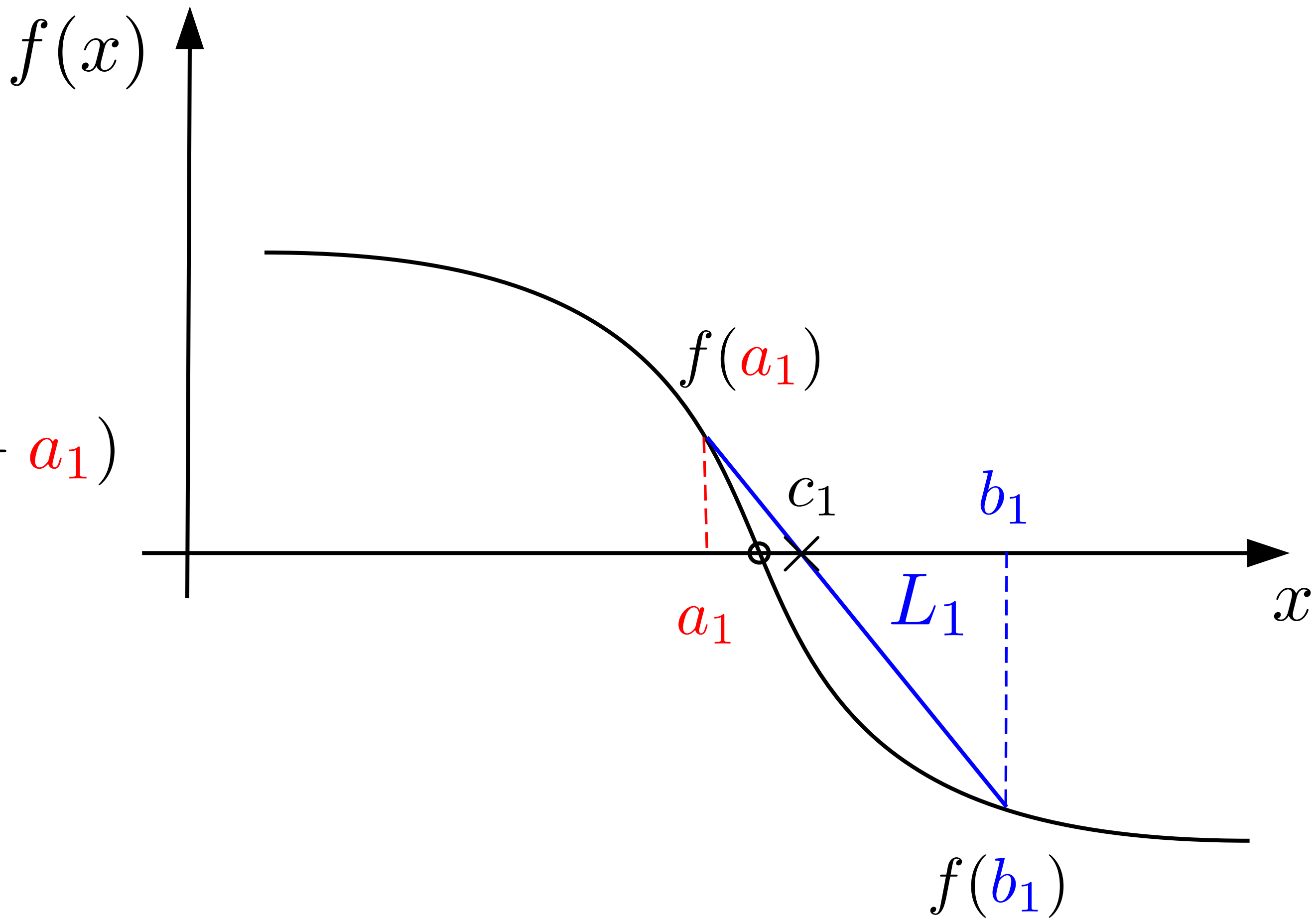
$$m = \frac{f(b_1) - f(a_1)}{b_1 - a_1}$$

$$L_1 : y = f(a_1) + \left(\frac{f(b_1) - f(a_1)}{b_1 - a_1}\right)(x - a_1)$$



Numerical Finding Root

- **False Position Method**



Step 1: The line intersects the x-axis and new point,

$$L_1 : y = f(a_1) + \left(\frac{f(b_1) - f(a_1)}{b_1 - a_1}\right)(x - a_1)$$

$$y(c_1) = 0$$

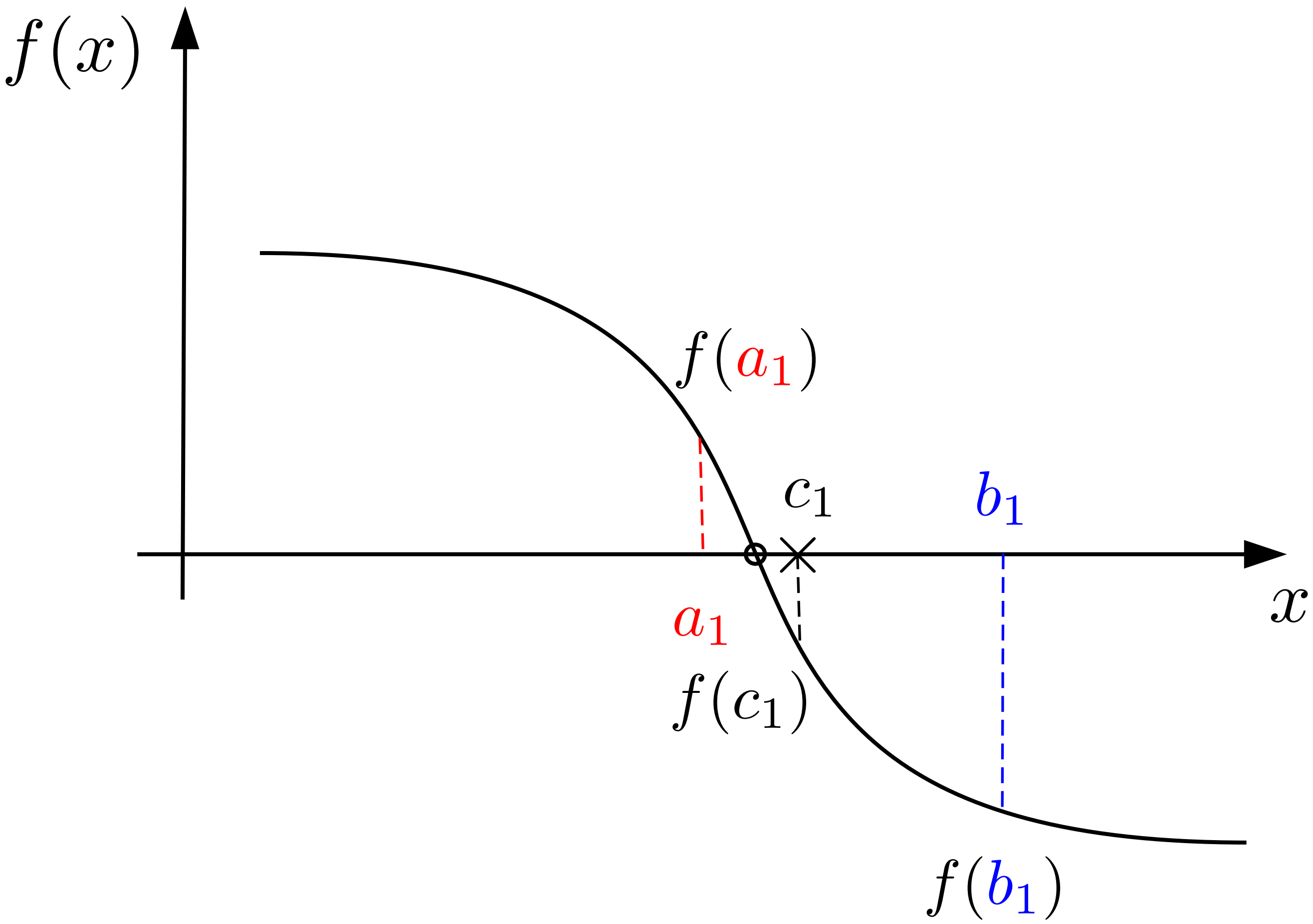
$$c_1 = a_1 - f(a_1)\left(\frac{b_1 - a_1}{f(b_1) - f(a_1)}\right)$$

Numerical Finding Root

- **False Position Method**

Step 2: Check if it has **converged** to a root within acceptable precision,

$$|f(c_1)| < \epsilon, \quad \epsilon \ll 1$$



Numerical Finding Root

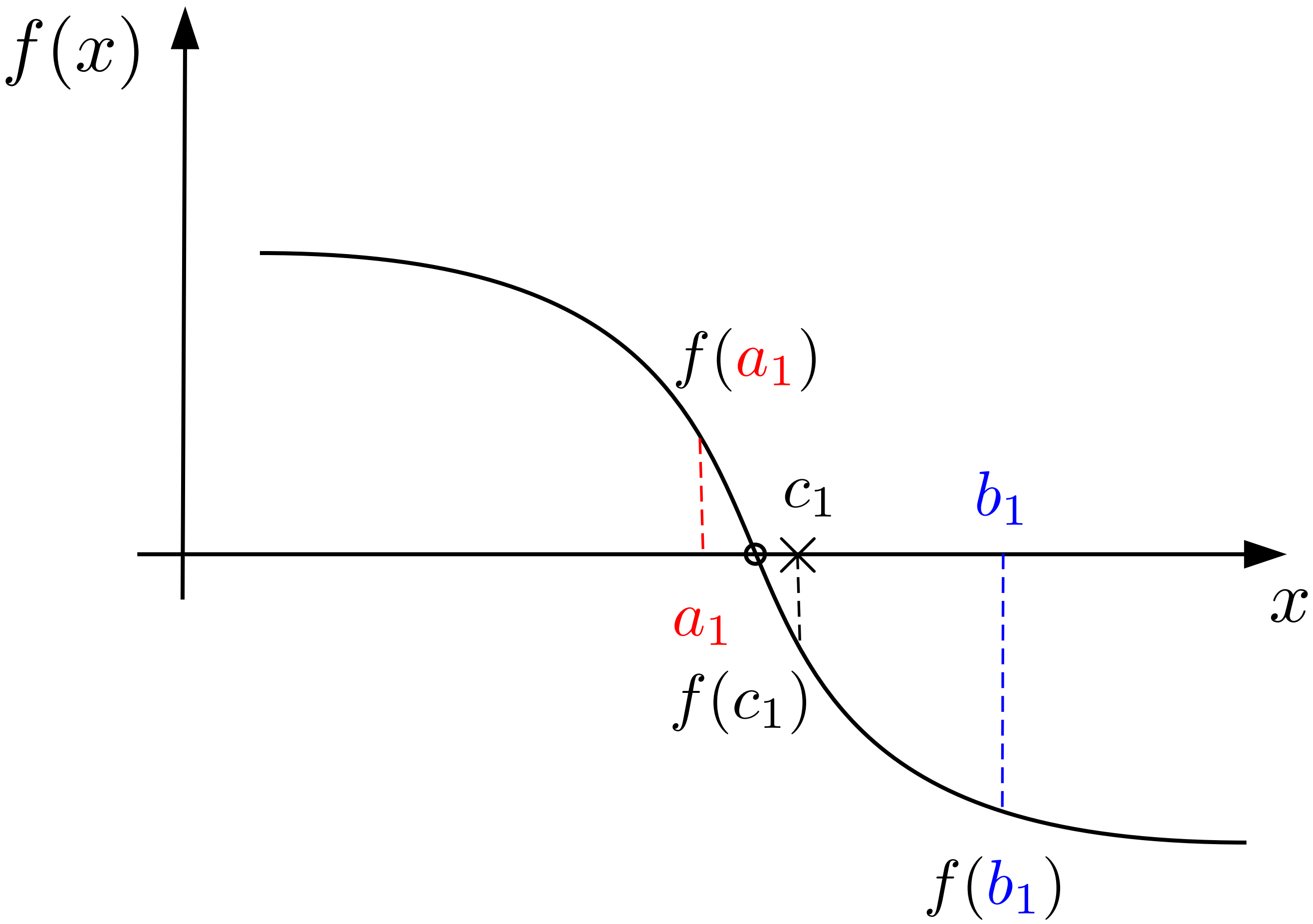
- **False Position Method**

Step 3:

New interval,

$$f(a_1)f(c_1) > 0$$

$$f(a_1)f(c_1) < 0$$



Numerical Finding Root

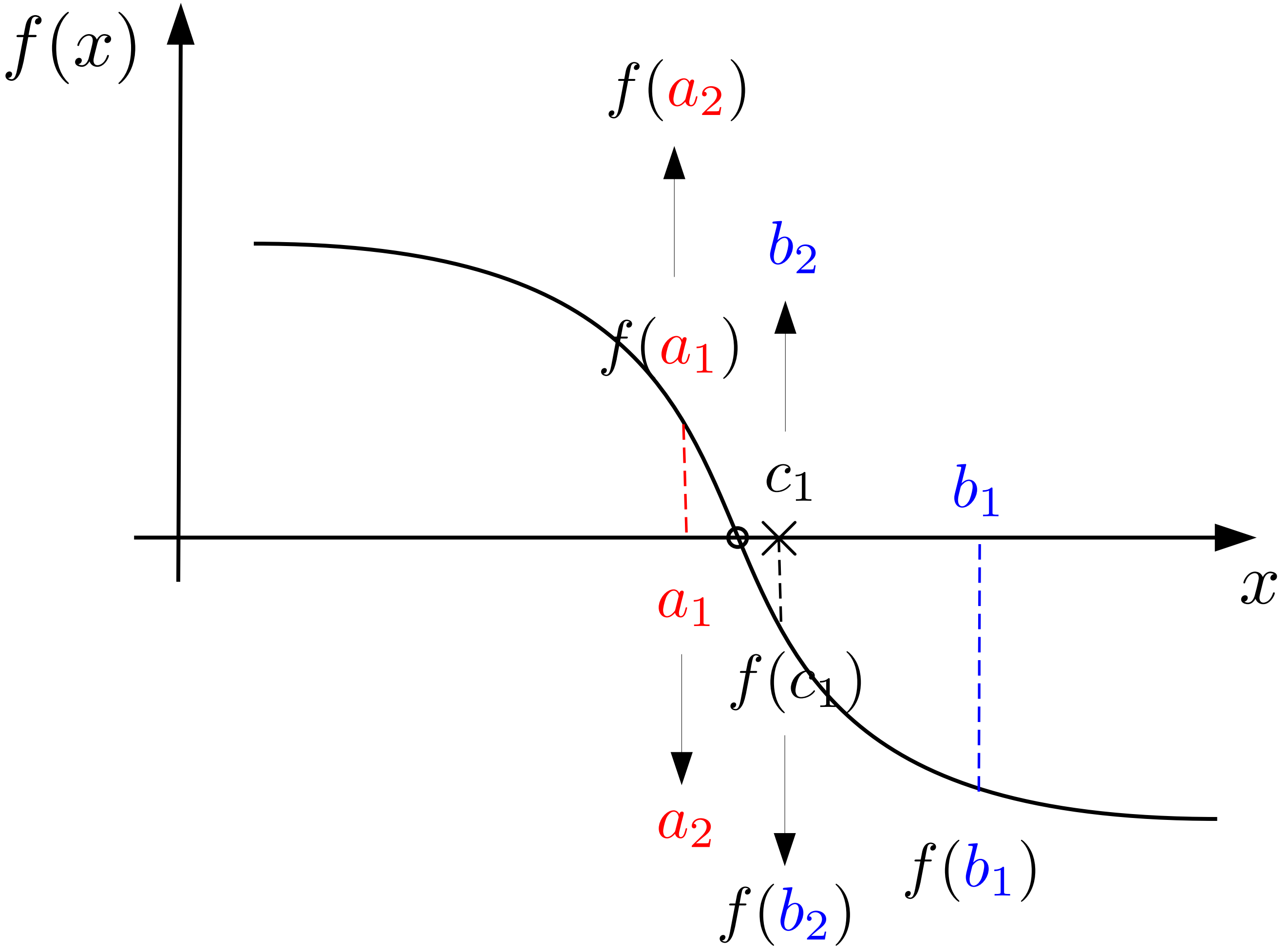
- **False Position Method**

Step 3:

New interval,

$$f(a_1)f(c_1) > 0$$
$$[a_2, b_2] = [c_1, b_1]$$

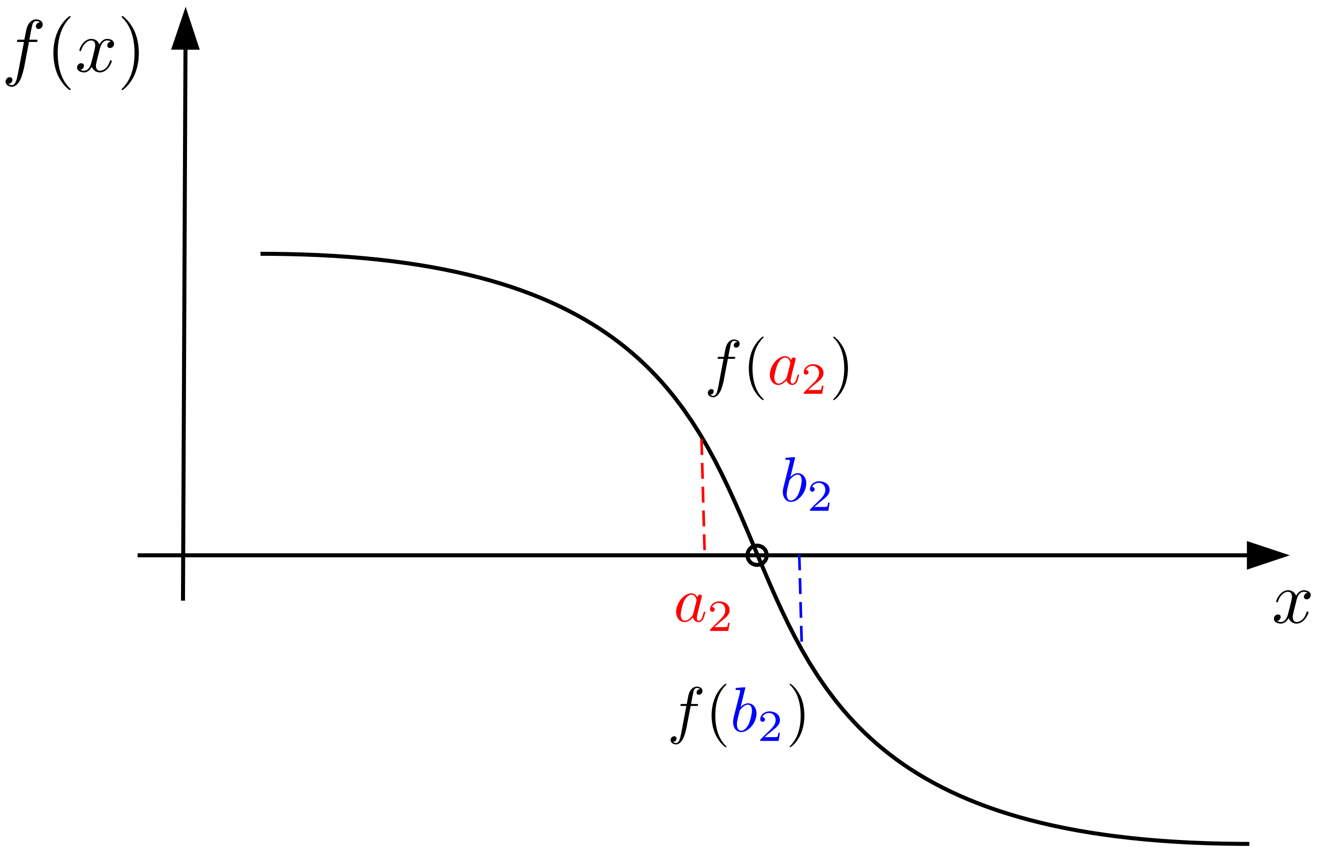
$$f(a_1)f(c_1) < 0$$
$$[a_2, b_2] = [a_1, c_1]$$



Numerical Finding Root

- **False Position Method**

Step 4: Return **step 1**



Numerical Finding Root

- **False Position Method**

Step 1: Calculate the **midpoint** of the interval

$$y(c_n) = 0$$

$$c_n = a_n - f(a_n)\left(\frac{b_n - a_n}{f(b_n) - f(a_n)}\right)$$

Step 2: Check if it has **converged** to a root within acceptable precision

$$|f(c_n)| < \epsilon, \quad \epsilon \ll 1$$

Step 3: **New interval**,

$$f(a_n)f(c_n) > 0$$

$$[a_{n+1}, b_{n+1}] = [c_n, b_n]$$

$$f(a_n)f(c_n) < 0$$

$$[a_{n+1}, b_{n+1}] = [a_n, c_n]$$

Step 4: Return to **Step 1**