Computational Physics

Lecture-15

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Contents

- Basis Concepts
- Numerical Differentiation
- Numerical Integration
- Numerical Finding Root
- Classical Scattering
- Solving Linear Systems

Solving Linear Systems

- **Gauss-Jordan Method (Gauss-Jordan Elimination)**

$$\begin{bmatrix} 1.0 & -1.0 & 2.0 & -1.0 \\ 2.0 & -2.0 & 3.0 & -3.0 \\ 1.0 & 1.0 & 1.0 & 0.0 \\ 1.0 & -1.0 & 4.0 & 3.0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8.0 \\ -20.0 \\ -2.0 \\ 4.0 \end{bmatrix}$$

The basic idea of Gauss-Jordan method is to transform the original linear equation set to one that has **unit** coefficient matrix.

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} 1.0 & -1.0 & 2.0 & -1.0 \\ 2.0 & -2.0 & 3.0 & -3.0 \\ 1.0 & 1.0 & 1.0 & 0.0 \\ 1.0 & -1.0 & 4.0 & 3.0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8.0 \\ -20.0 \\ -2.0 \\ 4.0 \end{bmatrix}$$

Gauss-Jordan

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\left[\begin{array}{cccc|c} 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right]$$

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \left[\begin{array}{cccc|c} 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right] \end{array} = \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \left[\begin{array}{cccc|c} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{array}\right] \end{array}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

Pivoting

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

Pivoting

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & a_{13}^{(3)} & a_{14}^{(3)} & a_{15}^{(3)} \\ 0 & 1 & a_{23}^{(3)} & a_{24}^{(3)} & a_{25}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & a_{35}^{(3)} \\ 0 & 0 & a_{43}^{(3)} & a_{44}^{(3)} & a_{45}^{(3)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & a_{13}^{(3)} & a_{14}^{(3)} & a_{15}^{(3)} \\ 0 & 1 & a_{23}^{(3)} & a_{24}^{(3)} & a_{25}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & a_{35}^{(3)} \\ 0 & 0 & a_{43}^{(3)} & a_{44}^{(3)} & a_{45}^{(3)} \end{bmatrix}$$

Pivoting

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & a_{14}^{(4)} & a_{15}^{(4)} \\ 0 & 1 & 0 & a_{24}^{(4)} & a_{25}^{(4)} \\ 0 & 0 & 1 & a_{34}^{(4)} & a_{35}^{(4)} \\ 0 & 0 & 0 & a_{45}^{(4)} & a_{45}^{(4)} \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 & a_{13}^{(3)} & a_{14}^{(3)} & a_{15}^{(3)} \\ 0 & 1 & a_{23}^{(3)} & a_{24}^{(3)} & a_{25}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & a_{35}^{(3)} \\ 0 & 0 & a_{43}^{(3)} & a_{44}^{(3)} & a_{45}^{(3)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & a_{13}^{(3)} & a_{14}^{(3)} & a_{15}^{(3)} \\ 0 & 1 & a_{23}^{(3)} & a_{24}^{(3)} & a_{25}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & a_{35}^{(3)} \\ 0 & 0 & a_{43}^{(3)} & a_{44}^{(3)} & a_{45}^{(3)} \end{bmatrix}$$

$$\leftarrow$$

Pivoting

$$\begin{bmatrix} 1 & 0 & 0 & a_{14}^{(4)} & a_{15}^{(4)} \\ 0 & 1 & 0 & a_{24}^{(4)} & a_{25}^{(4)} \\ 0 & 0 & 1 & a_{34}^{(4)} & a_{35}^{(4)} \\ 0 & 0 & 0 & a_{45}^{(4)} & a_{45}^{(4)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & a_{14}^{(1)} & a_{15}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & a_{24}^{(1)} & a_{25}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & a_{34}^{(1)} & a_{35}^{(1)} \\ a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(1)} & a_{44}^{(1)} & a_{45}^{(1)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & a_{12}^{(2)} & a_{13}^{(2)} & a_{14}^{(2)} & a_{15}^{(2)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & a_{24}^{(2)} & a_{25}^{(2)} \\ 0 & a_{32}^{(2)} & a_{33}^{(2)} & a_{34}^{(2)} & a_{35}^{(2)} \\ 0 & a_{42}^{(2)} & a_{43}^{(2)} & a_{44}^{(2)} & a_{45}^{(2)} \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & a_{13}^{(3)} & a_{14}^{(3)} & a_{15}^{(3)} \\ 0 & 1 & a_{23}^{(3)} & a_{24}^{(3)} & a_{25}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & a_{34}^{(3)} & a_{35}^{(3)} \\ 0 & 0 & a_{43}^{(3)} & a_{44}^{(3)} & a_{45}^{(3)} \end{bmatrix}$$

$$\leftarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & a_{14}^{(4)} & a_{15}^{(4)} \\ 0 & 1 & 0 & a_{24}^{(4)} & a_{25}^{(4)} \\ 0 & 0 & 1 & a_{34}^{(4)} & a_{35}^{(4)} \\ 0 & 0 & 0 & a_{45}^{(4)} & a_{45}^{(4)} \end{bmatrix}$$

$$\uparrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & a_{15}^{(5)} \\ 0 & 1 & 0 & 0 & a_{25}^{(5)} \\ 0 & 0 & 1 & 0 & a_{35}^{(5)} \\ 0 & 0 & 0 & 1 & a_{45}^{(5)} \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} 1 & 0 & 0 & 0 & a_{15}^{(5)} \\ 0 & 1 & 0 & 0 & a_{25}^{(5)} \\ 0 & 0 & 1 & 0 & a_{35}^{(5)} \\ 0 & 0 & 0 & 1 & a_{45}^{(5)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -7 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{bmatrix} 1.0 & -1.0 & 2.0 & -1.0 \\ 2.0 & -2.0 & 3.0 & -3.0 \\ 1.0 & 1.0 & 1.0 & 0.0 \\ 1.0 & -1.0 & 4.0 & 3.0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8.0 \\ -20.0 \\ -2.0 \\ 4.0 \end{bmatrix}$$

Gauss-Jordan

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

Solving Linear Systems

- **Gauss-Jordan Method**

Pivot Step

$$
\begin{array}{ccc}
\left[\begin{array}{cccc:c} 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right] & & \left[\begin{array}{cccc:c} 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right] \\
\uparrow & & \downarrow \\
\left[\begin{array}{cccc:c} 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right] & \leftarrow & \left[\begin{array}{cccc:c} 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 2.0 & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right]
\end{array}
$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\div \boxed{2.0} \Rightarrow \left[\begin{array}{cccc:c} \boxed{2.0} & -2.0 & 3.0 & -3.0 & -20.0 \\ 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \left[\begin{array}{cccc|c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 1.0 & -1.0 & 2.0 & -1.0 & -8.0 \\ 1.0 & 1.0 & 1.0 & 0.0 & -2.0 \\ 1.0 & -1.0 & 4.0 & 3.0 & 4.0 \end{array}\right]$$

$$\begin{matrix} R_1 \\ R_2 - 1.0 \times R_1 \\ R_3 - 1.0 \times R_1 \\ R_4 - 1.0 \times R_1 \end{matrix} \left[\begin{array}{cccc|c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 2.0 & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

Pivot Step

$$\left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 2.0 & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right] \quad \left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 2.0 & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

$$\uparrow \qquad\qquad\qquad\qquad \downarrow$$

$$\left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 2.0 & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right] \leftarrow \left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 2.0 & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\xrightarrow{\div\, 2.0}\left[\begin{array}{cccc|c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & \boxed{2.0} & -0.5 & 1.5 & 8.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{array}{r} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}\left[\begin{array}{cccc:c} 1.0 & -1.0 & 1.5 & -1.5 & -10.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

$$\begin{array}{r} R_1 - (-1) \times R_2 \\ R_2 \\ R_3 - 0 \times R_2 \\ R_4 - 0 \times R_2 \end{array}\left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

Pivot Step

$$\left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \end{array}\right] \quad \left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

$$\uparrow \qquad\qquad\qquad\qquad \downarrow$$

$$\left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right] \longleftarrow \left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \\ 0 & 0 & 2.5 & 4.5 & 14.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\div \boxed{2.5} \Rightarrow \left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & \boxed{2.5} & 4.5 & 14.0 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 1.0 & 1.8 & 5.6 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{matrix} \left[\begin{array}{cccc:c} 1.0 & 0.0 & 1.25 & -0.75 & -6.0 \\ 0 & 1.0 & -0.25 & 0.75 & 4.0 \\ 0 & 0 & 1.0 & 1.8 & 5.6 \\ 0 & 0 & 0.5 & 0.5 & 2.0 \end{array}\right]$$

$$\begin{matrix} R_1 - 1.25 \times R_3 \\ R_2 - (-0.25) \times R_3 \\ R_3 \\ R_4 - 0.5 \times R_3 \end{matrix} \left[\begin{array}{cccc:c} 1 & 0 & 0 & -3.0 & -13.0 \\ 0 & 1 & 0 & 1.2 & 5.4 \\ 0 & 0 & 1 & 1.8 & 5.6 \\ 0 & 0 & 0 & -0.4 & -0.8 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\xrightarrow{\div(-0.4)} \left[\begin{array}{cccc:c} 1 & 0 & 0 & -3.0 & -13.0 \\ 0 & 1 & 0 & 1.2 & 5.4 \\ 0 & 0 & 1 & 1.8 & 5.6 \\ 0 & 0 & 0 & \boxed{-0.4} & -0.8 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -3.0 & -13.0 \\ 0 & 1 & 0 & 1.2 & 5.4 \\ 0 & 0 & 1 & 1.8 & 5.6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -3.0 & -13.0 \\ 0 & 1 & 0 & 1.2 & 5.4 \\ 0 & 0 & 1 & 1.8 & 5.6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{array}{r} R_1 - (-3.0) \times R_4 \\ R_2 - 1.2 \times R_4 \\ R_3 - 1.8 \times R_4 \\ R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -7 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Solving Linear Systems

- **Gauss-Jordan Method**

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & -7 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 2 \\ 2 \end{bmatrix}$$

$$x_1 = -7, \quad x_2 = 3, \quad x_1 = 2, \quad x_4 = 2$$