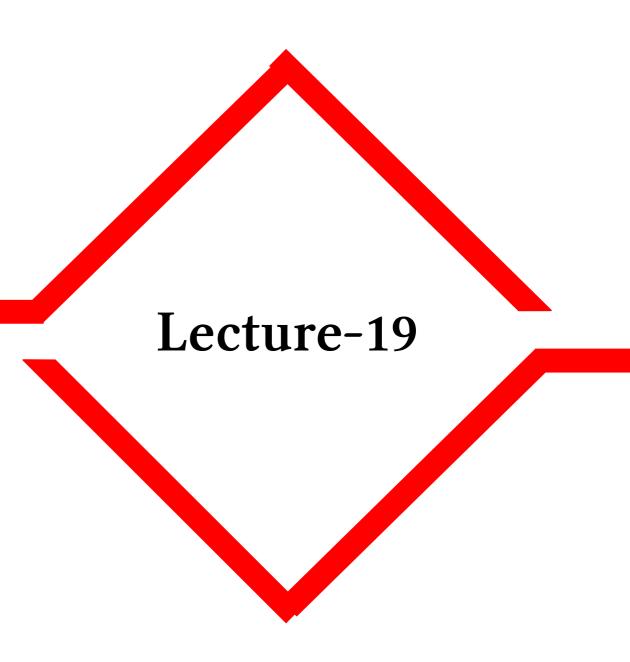
# Computational Physics

#### M. Reza Mozaffari

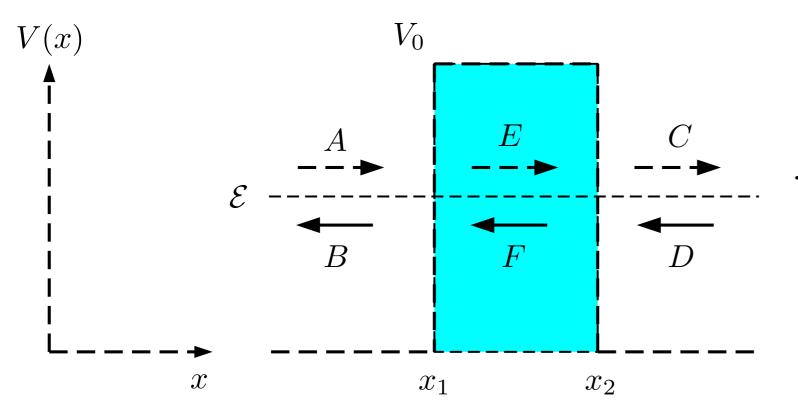
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- Basis Concepts
- Numerical Differentiation
- Numerical Integration
- Numerical Finding Root
- Classical Scattering
- Solving Linear Systems
- Transmission of Rectangular Barriers

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$



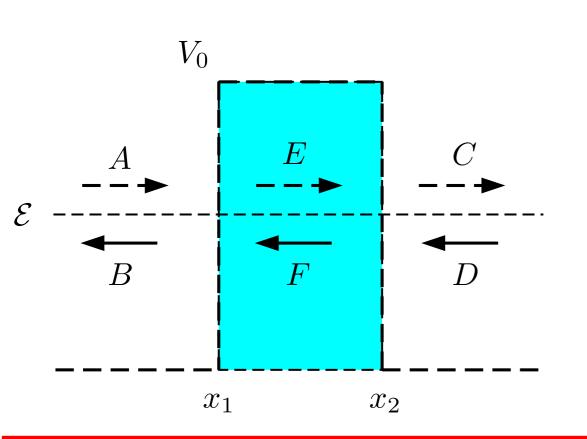
$$\begin{cases} -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{I}}}{\mathrm{d}x^2} = \mathcal{E}\psi_{\mathrm{I}} \\ -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{II}}}{\mathrm{d}x^2} + V_0 \psi_{\mathrm{II}} = \mathcal{E}\psi_{\mathrm{II}} \\ -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{III}}}{\mathrm{d}x^2} = \mathcal{E}\psi_{\mathrm{III}} \end{cases}$$

$$\begin{cases} \psi_{\rm I} = Ae^{ikx} + Be^{-ikx} \\ \psi_{\rm II} = Ee^{qx} + Fe^{-qx} \\ \psi_{\rm III} = Ce^{ikx} + De^{-ikx} \end{cases}$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(V_0 - \mathcal{E})}{\hbar^2}}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

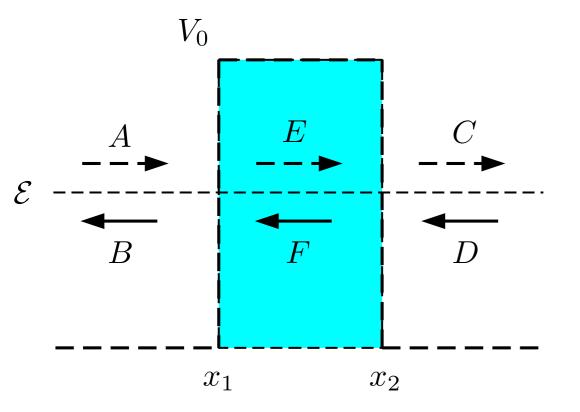
Transmission of Rectangular Barriers 
$$\begin{cases} \psi_{\rm I} = Ae^{ikx} + Be^{-ikx} \\ \psi_{\rm II} = Ee^{qx} + Fe^{-qx} \\ -\frac{\hbar^2}{2m}\frac{{\rm d}^2\psi}{{\rm d}x^2} + V(x)\psi = \mathcal{E}\psi \end{cases}$$
 
$$\psi_{\rm III} = Ce^{ikx} + De^{-ikx}$$



$$\begin{cases} \psi_{\rm I}(x_1) = \psi_{\rm II}(x_1) & \psi_{\rm II}(x_2) = \psi_{\rm III}(x_2) \\ \frac{\mathrm{d}\psi_{\rm I}}{\mathrm{d}x}|_{x_1} = \frac{\mathrm{d}\psi_{\rm II}}{\mathrm{d}x}|_{x_1} & \frac{\mathrm{d}\psi_{\rm II}}{\mathrm{d}x}|_{x_2} = \frac{\mathrm{d}\psi_{\rm III}}{\mathrm{d}x}|_{x_2} \end{cases}$$

$$\begin{cases} Ae^{ikx_1} + Be^{-ikx_1} = Ee^{qx_1} + Fe^{-qx_1} \\ ik(Ae^{ikx_1} - Be^{-ikx_1}) = q(Ee^{qx_1} - Fe^{-qx_1}) \\ Ee^{qx_2} + Fe^{-qx_2} = Ce^{ikx_2} + De^{-ikx_2} \\ q(Ee^{qx_2} - Fe^{-qx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2}) \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$



$$\begin{cases} Ae^{ikx_1} + Be^{-ikx_1} = Ee^{qx_1} + Fe^{-qx_1} \\ ik(Ae^{ikx_1} - Be^{-ikx_1}) = q(Ee^{qx_1} - Fe^{-qx_1}) \end{cases}$$

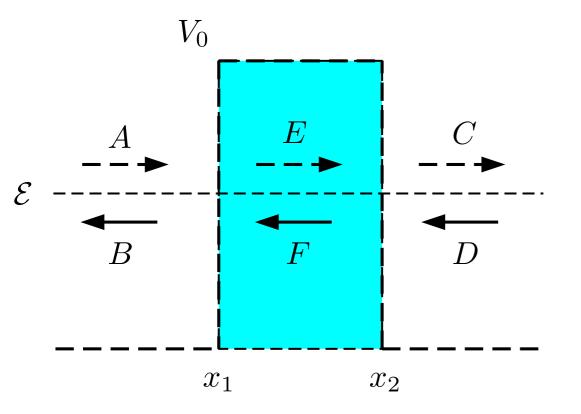
$$Ee^{qx_2} + Fe^{-qx_2} = Ce^{ikx_2} + De^{-ikx_2}$$

$$q(Ee^{qx_2} - Fe^{-qx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2})$$

$$\begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} =$$

$$\begin{bmatrix} e^{qx_1} & e^{-qx_1} \\ qe^{qx_1} & -qe^{-qx_1} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$



$$\begin{cases} Ae^{ikx_1} + Be^{-ikx_1} = Ee^{qx_1} + Fe^{-qx_1} \\ ik(Ae^{ikx_1} - Be^{-ikx_1}) = q(Ee^{qx_1} - Fe^{-qx_1}) \\ Ee^{qx_2} + Fe^{-qx_2} = Ce^{ikx_2} + De^{-ikx_2} \\ q(Ee^{qx_2} - Fe^{-qx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2}) \end{cases}$$

$$\begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} e^{qx_1} & e^{-qx_1} \\ qe^{qx_1} & -qe^{-qx_1} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix}$$

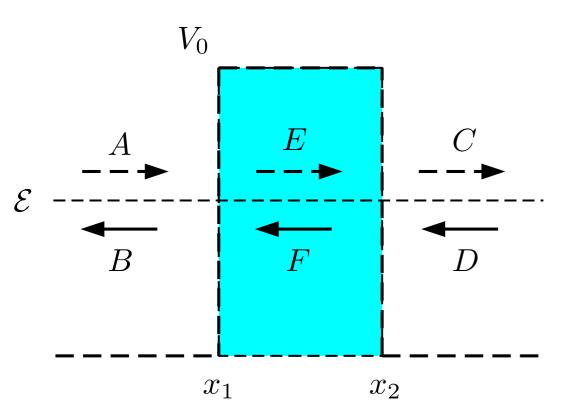
$$\begin{bmatrix} e^{qx_2} & e^{-qx_2} \\ qe^{qx_2} & -qe^{-qx_2} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} = \begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

# Transmission of Rectangular Barriers $M_1 = \begin{bmatrix} e^{i\kappa x_1} & e^{-i\kappa x_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$

$$M_1 = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_2 = \begin{bmatrix} e^{qx_1} & e^{-qx_1} \\ qe^{qx_1} & -qe^{-qx_1} \end{bmatrix}$$



$$M_3 = \begin{bmatrix} e^{qx_2} & e^{-qx_2} \\ qe^{qx_2} & -qe^{-qx_2} \end{bmatrix}$$

$$M_4 = \begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix}$$

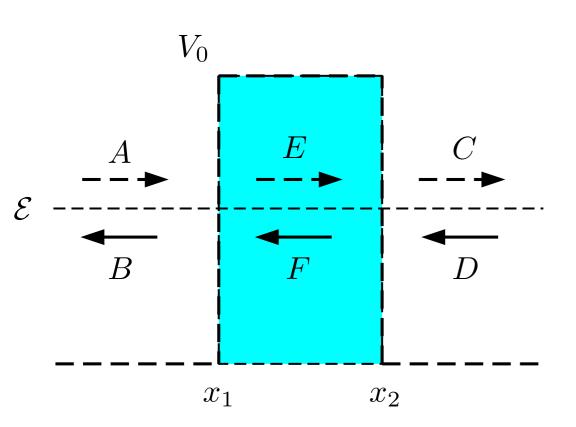
$$M_1 \left[ egin{array}{c} A \ B \end{array} 
ight] = M_2 \left[ egin{array}{c} E \ F \end{array} 
ight]$$

$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$M_1 \left[ egin{array}{c} A \ B \end{array} 
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$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$



$$M_2^{-1}M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = \left[ \begin{array}{c} E \\ F \end{array} \right]$$

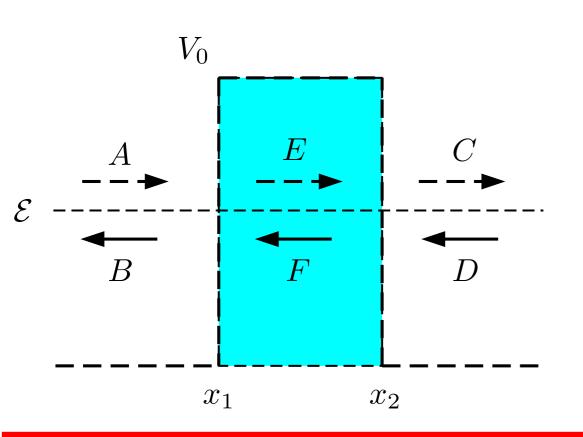
$$M_4^{-1}M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = \left[ \begin{array}{c} C \\ D \end{array} \right]$$

## **Transmission of Rectangular Barriers** $M_1 \begin{vmatrix} A \\ B \end{vmatrix} = M_2 \begin{vmatrix} E \\ F \end{vmatrix}$

$$M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = M_2 \left[ \begin{array}{c} E \\ F \end{array} \right]$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$



$$M_2^{-1}M_1 \begin{bmatrix} A \\ B \end{bmatrix} = \left| \begin{bmatrix} E \\ F \end{bmatrix} \right|$$

$$M_4^{-1}M_3 \left| \begin{bmatrix} E \\ F \end{bmatrix} \right| = \begin{bmatrix} C \\ D \end{bmatrix}$$

$$M_4^{-1}M_3M_2^{-1}M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$\mathcal{M} = M_4^{-1} M_3 M_2^{-1} M_1, \qquad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}$$

# Transmission of Rectangular Barriers $M_1 = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$\mathcal{E} \xrightarrow{A} \qquad \mathcal{E} \qquad \mathcal{$$

$$M_1 = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} e^{qx_1} & e^{-qx_1} \\ qe^{qx_1} & -qe^{-qx_1} \end{bmatrix}$$

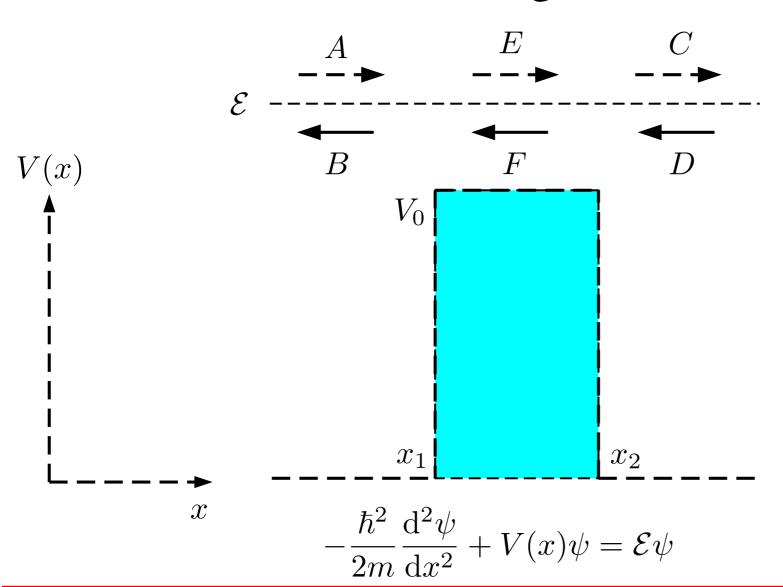
$$M_3 = \begin{bmatrix} e^{qx_2} & e^{-qx_2} \\ qe^{qx_2} & -qe^{-qx_2} \end{bmatrix}$$

$$M_4 = \begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix}$$

$$\mathcal{M} = M_4^{-1} M_3 M_2^{-1} M_1, \qquad \left[ \begin{array}{c} C \\ D \end{array} \right] = \mathcal{M} \left[ \begin{array}{c} A \\ B \end{array} \right]$$

$$\mathcal{M} = \mathcal{M}(V_0, \mathcal{E}, x_1, x_2)$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(V_0 - \mathcal{E})}{\hbar^2}}$$



$$\begin{cases} -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{I}}}{\mathrm{d}x^2} = \mathcal{E}\psi_{\mathrm{I}} \\ -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{II}}}{\mathrm{d}x^2} + V_0 \psi_{\mathrm{II}} = \mathcal{E}\psi_{\mathrm{II}} \\ -\frac{\hbar^2}{2m} \frac{\mathrm{d}^2 \psi_{\mathrm{III}}}{\mathrm{d}x^2} = \mathcal{E}\psi_{\mathrm{III}} \end{cases}$$

$$\begin{cases} \psi_{\rm I} = Ae^{ikx} + Be^{-ikx} \\ \psi_{\rm II} = Ee^{iqx} + Fe^{-iqx} \end{cases}$$

$$\psi_{\rm III} = Ce^{ikx} + De^{-ikx}$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(\mathcal{E} - V_0)}{\hbar^2}}$$

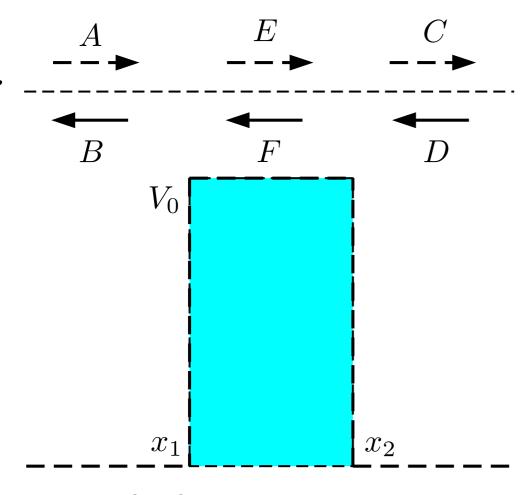
Transmission of Rectangular Barriers 
$$\begin{cases} \psi_{\rm I} = Ae^{ikx} + Be^{-ikx} \\ \psi_{\rm II} = Ee^{iqx} + Fe^{-iqx} \\ \psi_{\rm III} = Ce^{ikx} + De^{-ikx} \end{cases}$$

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$$\begin{cases} \psi_{\mathrm{I}}(x_1) = \psi_{\mathrm{II}}(x_1) & \begin{cases} \psi_{\mathrm{II}}(x_2) = \psi_{\mathrm{III}}(x_2) \\ \frac{\mathrm{d}\psi_{\mathrm{I}}}{\mathrm{d}x}|_{x_1} = \frac{\mathrm{d}\psi_{\mathrm{II}}}{\mathrm{d}x}|_{x_1} & \begin{cases} \frac{\mathrm{d}\psi_{\mathrm{II}}}{\mathrm{d}x}|_{x_2} = \frac{\mathrm{d}\psi_{\mathrm{III}}}{\mathrm{d}x}|_{x_2} \end{cases}$$

$$\begin{cases} Ae^{ikx_1} + Be^{-ikx_1} = Ee^{iqx_1} + Fe^{-iqx_1} \\ ik(Ae^{ikx_1} - Be^{-ikx_1}) = iq(Ee^{iqx_1} - Fe^{-iqx_1}) \\ Ee^{iqx_2} + Fe^{-iqx_2} = Ce^{ikx_2} + De^{-ikx_2} \\ iq(Ee^{iqx_2} - Fe^{-iqx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2}) \end{cases}$$

$$iq(Ee^{iqx_2} - Fe^{-iqx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2})$$



$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$\begin{cases} Ae^{ikx_1} + Be^{-ikx_1} = Ee^{iqx_1} + Fe^{-iqx_1} \\ ik(Ae^{ikx_1} - Be^{-ikx_1}) = iq(Ee^{iqx_1} - Fe^{-iqx_1}) \\ Ee^{iqx_2} + Fe^{-iqx_2} = Ce^{ikx_2} + De^{-ikx_2} \\ iq(Ee^{iqx_2} - Fe^{-iqx_2}) = ik(Ce^{ikx_2} - De^{-ikx_2}) \\ \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \\ \begin{bmatrix} e^{iqx_1} & e^{-iqx_1} \\ iqe^{iqx_1} & -iqe^{-iqx_1} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} \end{cases}$$

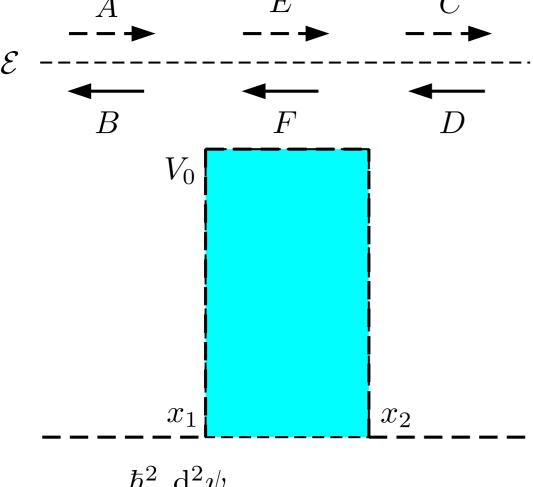
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$$\begin{bmatrix} e^{iqx_2} & e^{-iqx_2} \\ iqe^{iqx_2} & -iqe^{-iqx_2} \end{bmatrix} \begin{bmatrix} E \\ F \end{bmatrix} =$$

$$\mathcal{E}\psi$$

$$\begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

Transmission of Rectangular Barriers 
$$M_1 = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$$



$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_2 = \begin{bmatrix} e^{iqx_1} & e^{-iqx_1} \\ iqe^{iqx_1} & -iqe^{-iqx_1} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} e^{iqx_2} & e^{-iqx_2} \\ iqe^{iqx_2} & -iqe^{-iqx_2} \end{bmatrix}$$

$$M_4 = \begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix}$$

$$M_1 \left[ egin{array}{c} A \\ B \end{array} 
ight] = M_2 \left[ egin{array}{c} E \\ F \end{array} 
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$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$

# Transmission of Rectangular Barriers $M_1 \begin{vmatrix} A \\ B \end{vmatrix} = M_2 \begin{vmatrix} E \\ F \end{vmatrix}$

$$M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = M_2 \left[ \begin{array}{c} E \\ F \end{array} \right]$$

$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$F$$
  $D$   $V_0$   $x_1$   $x_2$ 

$$M_2^{-1}M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = \left[ \begin{array}{c} E \\ F \end{array} \right]$$

$$M_4^{-1}M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = M_2 \left[ \begin{array}{c} E \\ F \end{array} \right]$$

$$M_3 \left[ \begin{array}{c} E \\ F \end{array} \right] = M_4 \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$F$$
  $D$   $V_0$   $x_1$   $x_2$ 

$$M_2^{-1}M_1 \begin{bmatrix} A \\ B \end{bmatrix} = \left[ \begin{bmatrix} E \\ F \end{bmatrix} \right]$$

$$M_4^{-1}M_3 \left[ \begin{bmatrix} E \\ F \end{bmatrix} \right] = \begin{bmatrix} C \\ D \end{bmatrix}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_4^{-1}M_3M_2^{-1}M_1 \left[ \begin{array}{c} A \\ B \end{array} \right] = \left[ \begin{array}{c} C \\ D \end{array} \right]$$

$$\mathcal{M} = M_4^{-1} M_3 M_2^{-1} M_1, \qquad \left| \begin{array}{c} C \\ D \end{array} \right| = \mathcal{M} \left| \begin{array}{c} A \\ B \end{array} \right|$$

$$\left[\begin{array}{c} C \\ D \end{array}\right] = \mathcal{M} \left[\begin{array}{c} A \\ B \end{array}\right]$$

# Transmission of Rectangular Barriers $M_1 = \begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1} \end{bmatrix}$

$$\mathcal{E} \xrightarrow{A} \xrightarrow{E} \xrightarrow{C}$$

$$B \xrightarrow{F} D$$

$$V_0 \xrightarrow{X_1} x_2$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$

$$M_2 = \begin{bmatrix} e^{iqx_1} & e^{-iqx_1} \\ iqe^{iqx_1} & -iqe^{-iqx_1} \end{bmatrix}$$

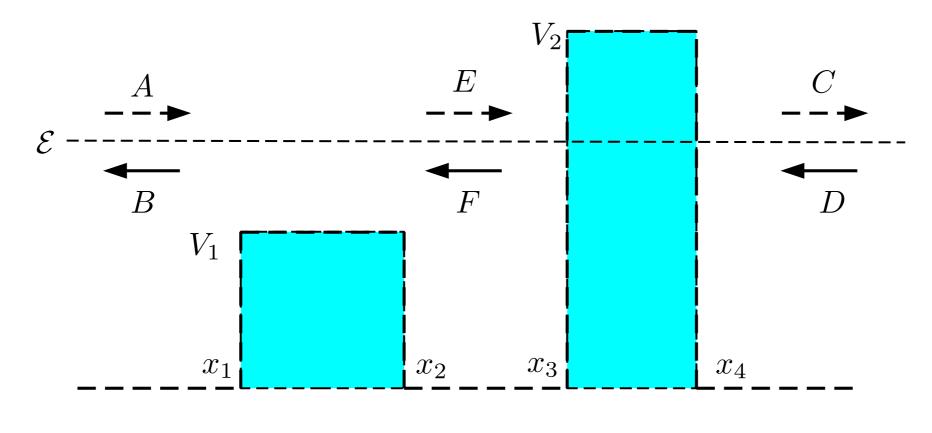
$$M_3 = \begin{bmatrix} e^{iqx_2} & e^{-iqx_2} \\ iqe^{iqx_2} & -iqe^{-iqx_2} \end{bmatrix}$$

$$M_4 = \begin{bmatrix} e^{ikx_2} & e^{-ikx_2} \\ ike^{ikx_2} & -ike^{-ikx_2} \end{bmatrix}$$

$$\mathcal{M} = M_4^{-1} M_3 M_2^{-1} M_1, \qquad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}$$

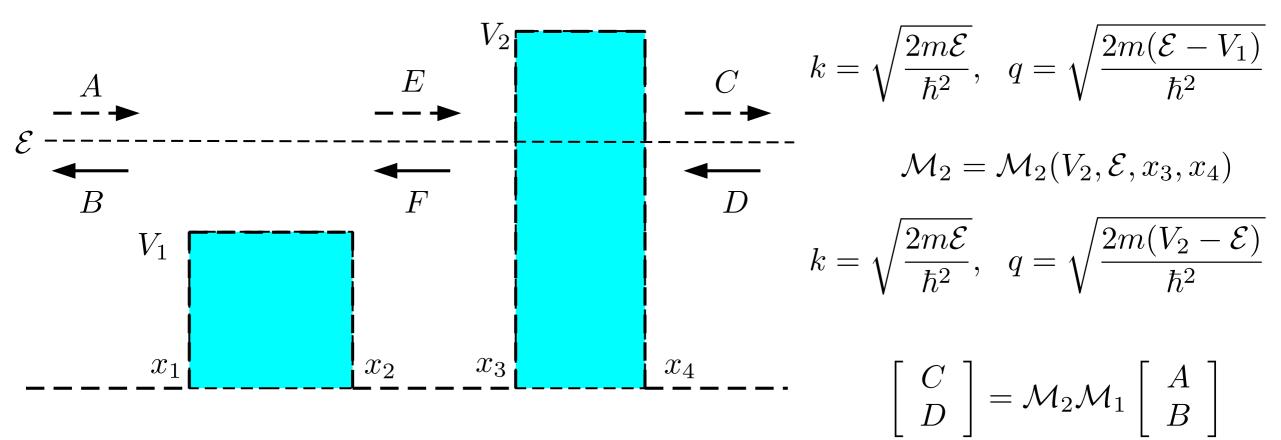
$$\mathcal{M} = \mathcal{M}(V_0, \mathcal{E}, x_1, x_2)$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(\mathcal{E} - V_0)}{\hbar^2}}$$



$$\left[\begin{array}{c} E \\ F \end{array}\right] = \mathcal{M}_1 \left[\begin{array}{c} A \\ B \end{array}\right] \quad \left[\begin{array}{c} C \\ D \end{array}\right] = \mathcal{M}_2 \left[\begin{array}{c} E \\ F \end{array}\right]$$

$$\mathcal{M}_1 = \mathcal{M}_1(V_1, \mathcal{E}, x_1, x_2)$$



$$\begin{bmatrix} E \\ F \end{bmatrix} = \mathcal{M}_1 \begin{bmatrix} A \\ B \end{bmatrix} \quad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M}_2 \begin{bmatrix} E \\ F \end{bmatrix} \qquad \mathcal{M} = \mathcal{M}_2 \mathcal{M}_1, \quad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\mathcal{M} = \mathcal{M}_2 \mathcal{M}_1, \quad \left[ egin{array}{c} C \ D \end{array} 
ight] = \mathcal{M} \left[ egin{array}{c} A \ B \end{array} 
ight]$$

$$\left[\begin{array}{c} C \\ D \end{array}\right] = \mathcal{M} \left[\begin{array}{c} A \\ B \end{array}\right], \quad D = 0$$

$$\left[\begin{array}{c} C \\ 0 \end{array}\right] = \left[\begin{array}{cc} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{array}\right] \left[\begin{array}{c} A \\ B \end{array}\right]$$

$$\mathcal{M}_{21}A + \mathcal{M}_{22}B = 0 \Rightarrow \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$
 
$$C = \mathcal{M}_{11}A + \mathcal{M}_{12}B \Rightarrow \frac{C}{A} = \mathcal{M}_{11} + \mathcal{M}_{12}\frac{B}{A}$$

$$r = \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$R = |r|^2 = rr^* = \left| \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}} \right|^2$$

$$t = \frac{C}{\Delta} = \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$T = |t|^2 = tt^* = \left| \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}} \right|^2$$

$$\left[\begin{array}{c} C \\ D \end{array}\right] = \mathcal{M} \left[\begin{array}{c} A \\ B \end{array}\right], \quad D = 0$$

$$\left[\begin{array}{c} C \\ 0 \end{array}\right] = \left[\begin{array}{cc} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{array}\right] \left[\begin{array}{c} A \\ B \end{array}\right]$$

$$r = \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$R = \left|r\right|^2 = rr^* = \left|\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}\right|^2$$

$$t = \frac{C}{A} = \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$T = |t|^2 = tt^* = \left| \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}} \right|^2$$

$$T(\mathcal{E}) + R(\mathcal{E}) = 1$$

