Computational Physics

Lecture-19

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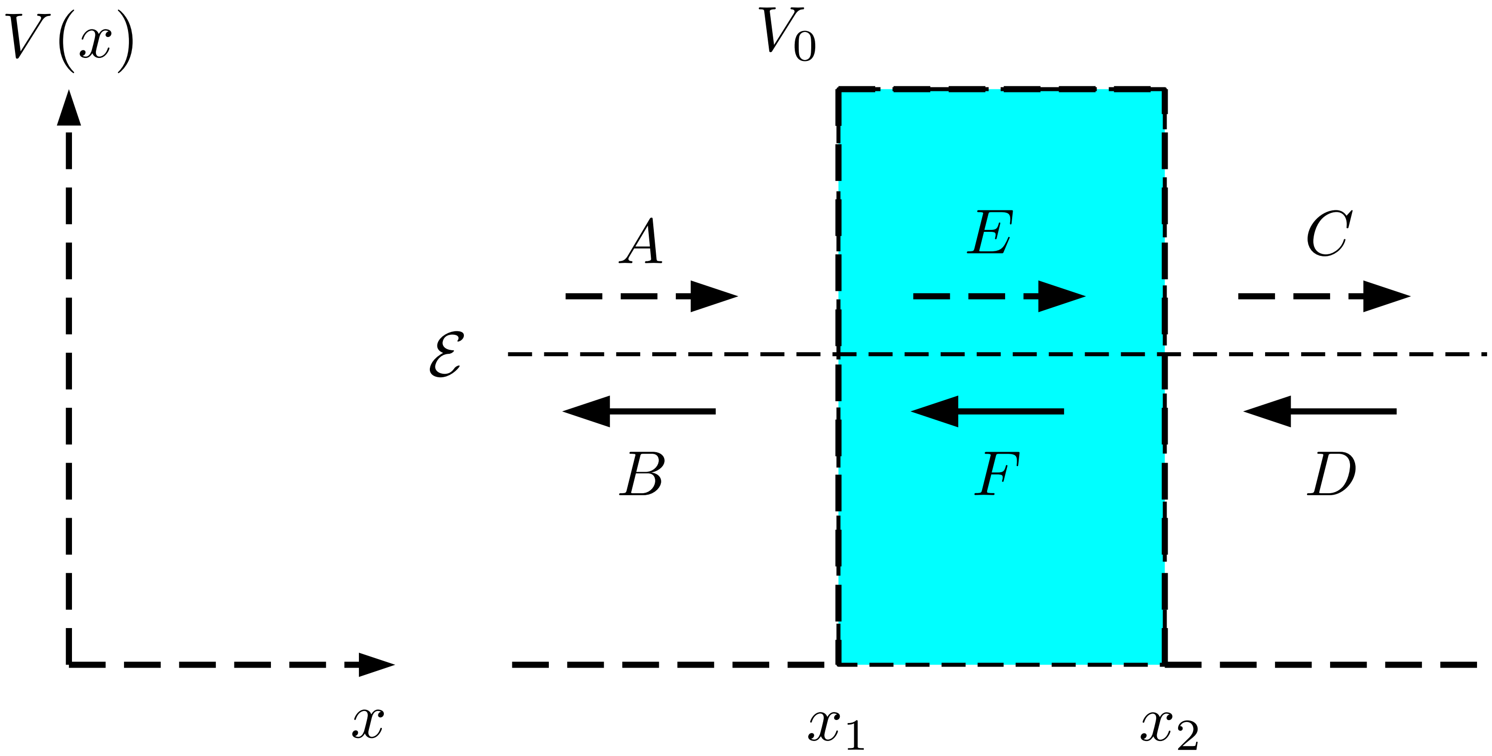
Physics Group, University of Qom

Contents

Transmission of Rectangular Barriers

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+V(x)\psi=\mathcal{E}\psi$$

$$\begin{cases} -\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2\psi_{\mathrm{I}}}{\mathrm{d}x^2}=\mathcal{E}\psi_{\mathrm{I}} \\ -\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2\psi_{\mathrm{II}}}{\mathrm{d}x^2}+V_0\psi_{\mathrm{II}}=\mathcal{E}\psi_{\mathrm{II}} \\ -\dfrac{\hbar^2}{2m}\dfrac{\mathrm{d}^2\psi_{\mathrm{III}}}{\mathrm{d}x^2}=\mathcal{E}\psi_{\mathrm{III}} \end{cases}$$



$$\begin{cases} \psi_{\mathrm{I}}=Ae^{ikx}+Be^{-ikx} \\ \psi_{\mathrm{II}}=Ee^{qx}+Fe^{-qx} \\ \psi_{\mathrm{III}}=Ce^{ikx}+De^{-ikx} \end{cases}$$

$$k=\sqrt{\frac{2m\mathcal{E}}{\hbar^2}},\quad q=\sqrt{\frac{2m(V_0-\mathcal{E})}{\hbar^2}}$$

Transmission of Rectangular Barriers

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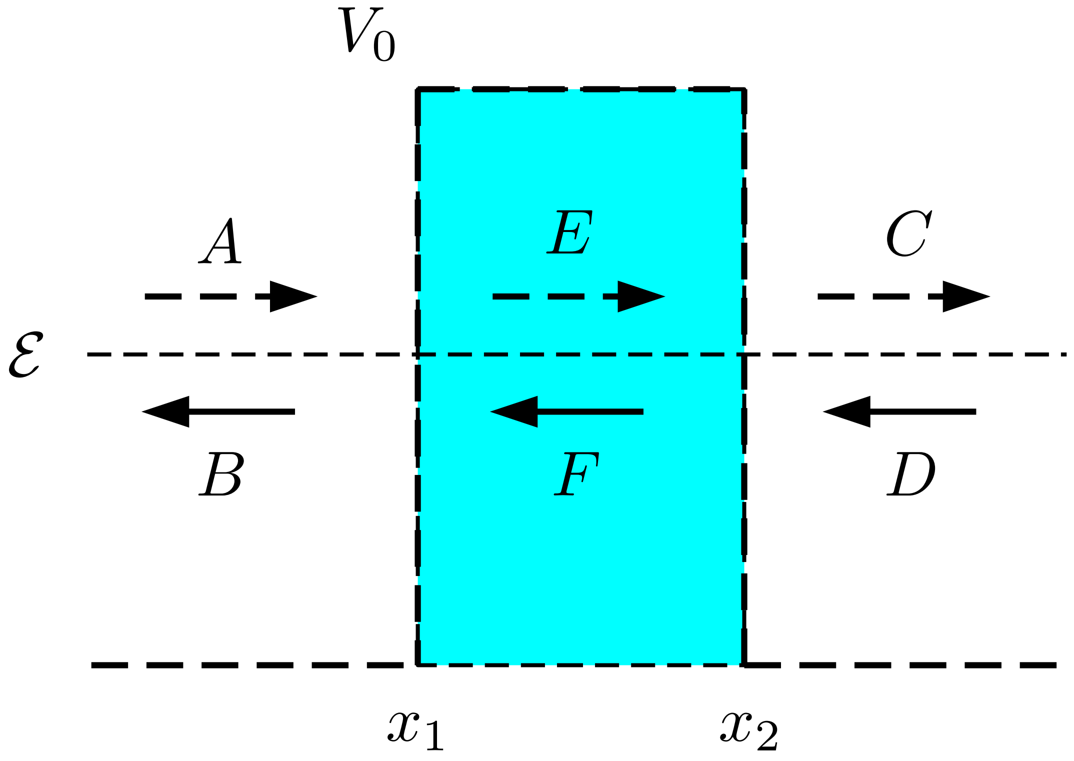

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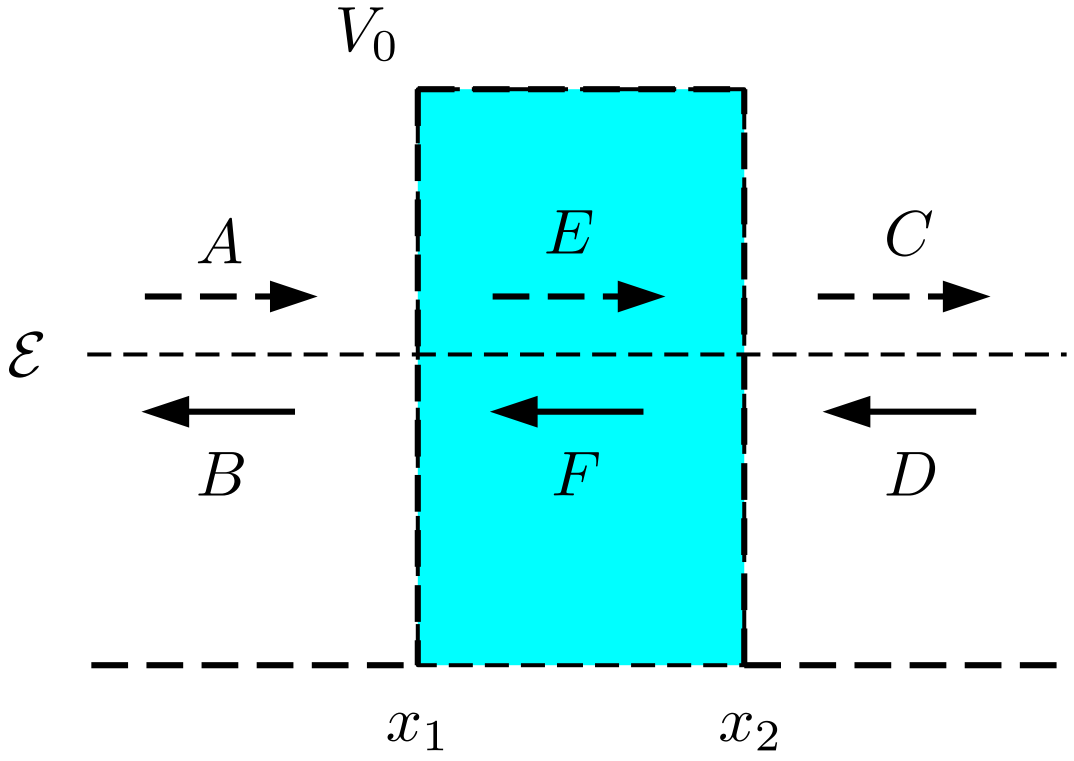
Transmission of Rect...

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + V(x)\psi = \mathcal{E}\psi$$



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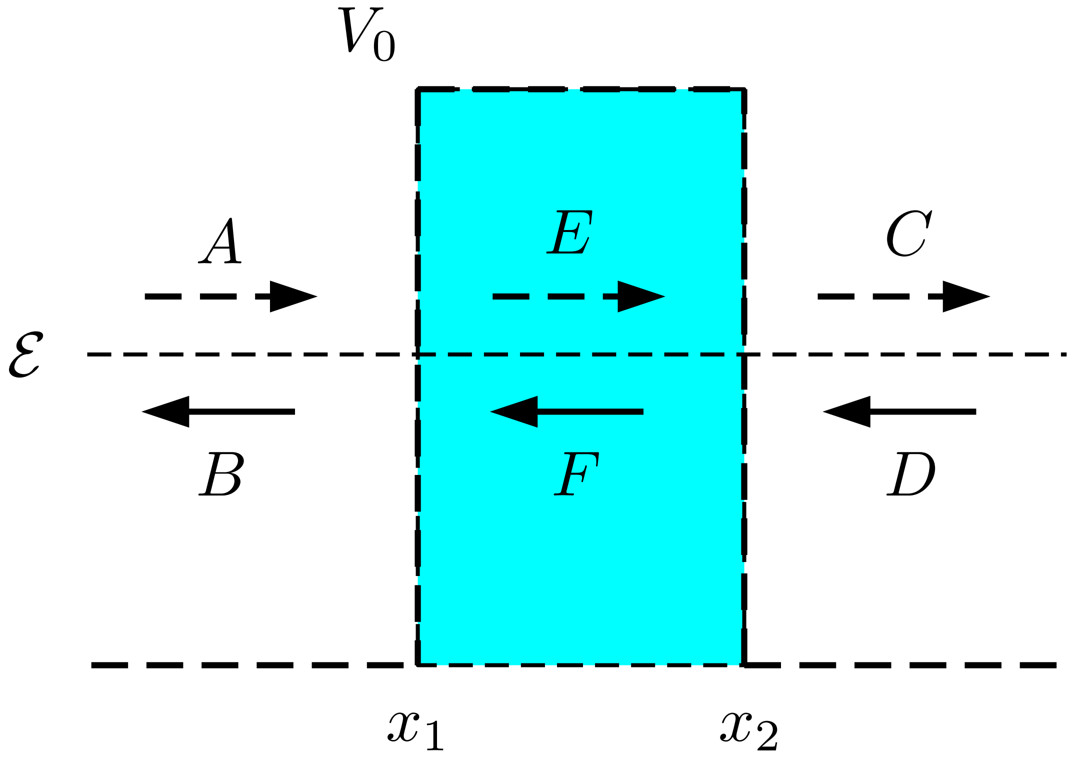

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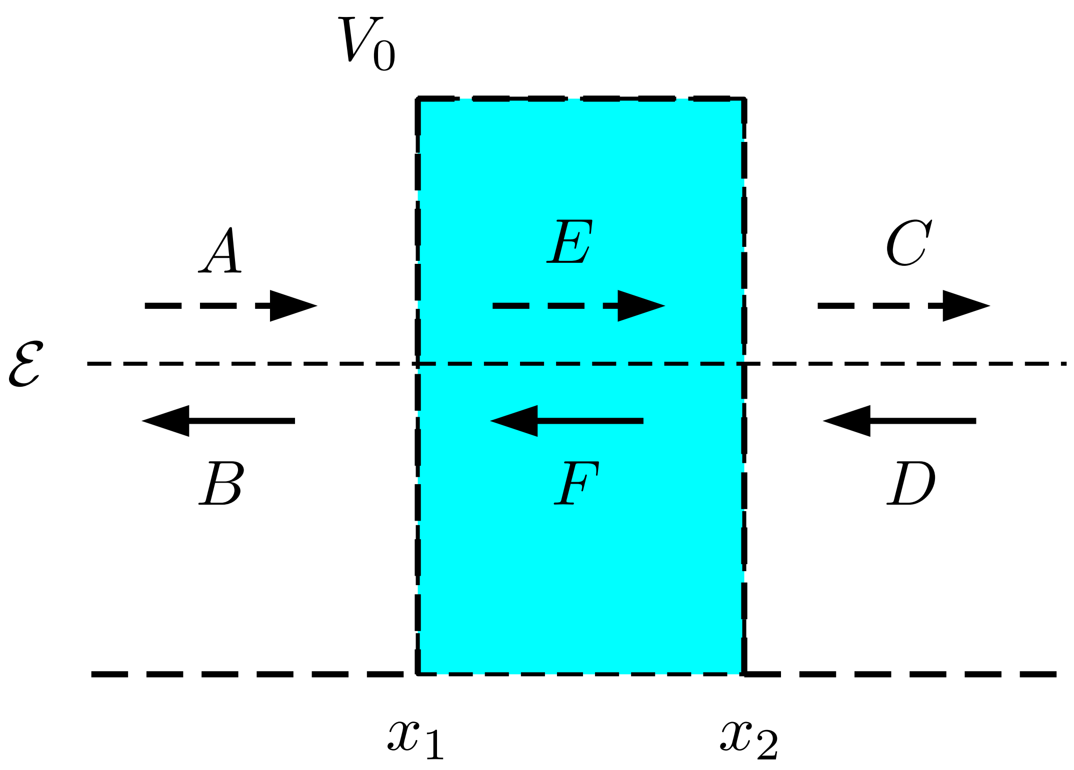
Transmission of Rectangular Barriers

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+V(x)\psi=\mathcal{E}\psi$$



$$M_1=\begin{bmatrix} e^{ikx_1} & e^{-ikx_1} \\ ike^{ikx_1} & -ike^{-ikx_1}\end{bmatrix}$$

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$$M_1\begin{bmatrix} A \\ B\end{bmatrix}=M_2\begin{bmatrix} E \\ F\end{bmatrix}$$

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Transmission of Rectangular Barriers

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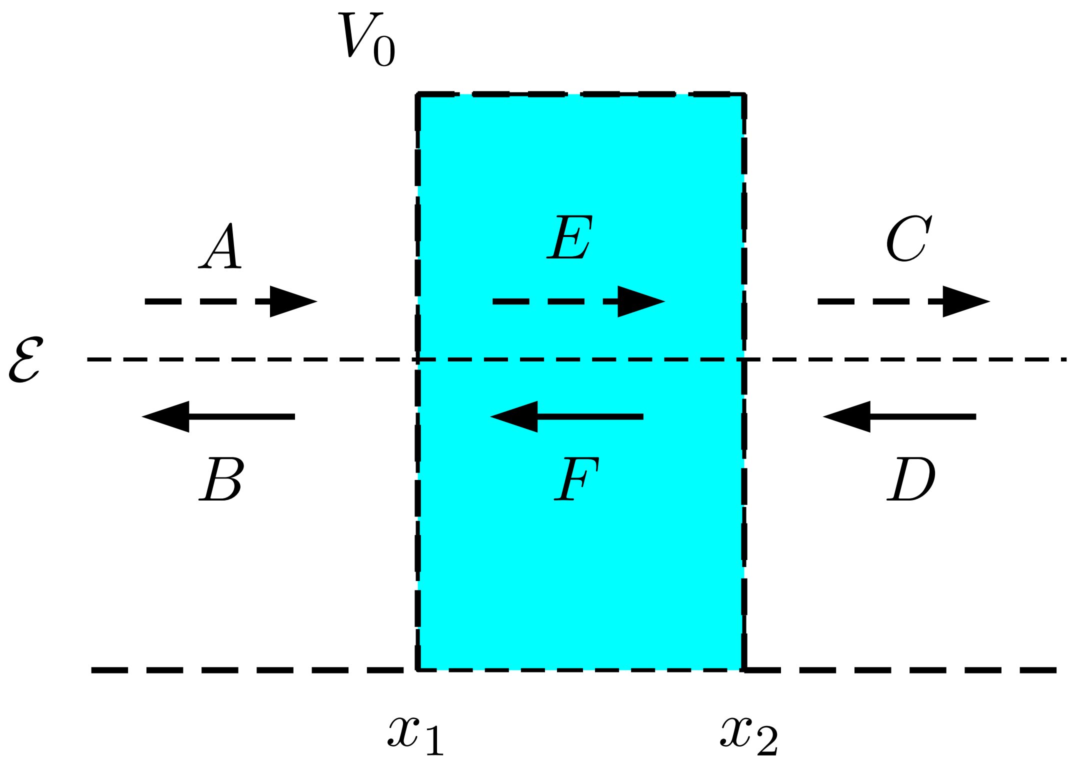

$$M_2^{-1} M_1 \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}$$

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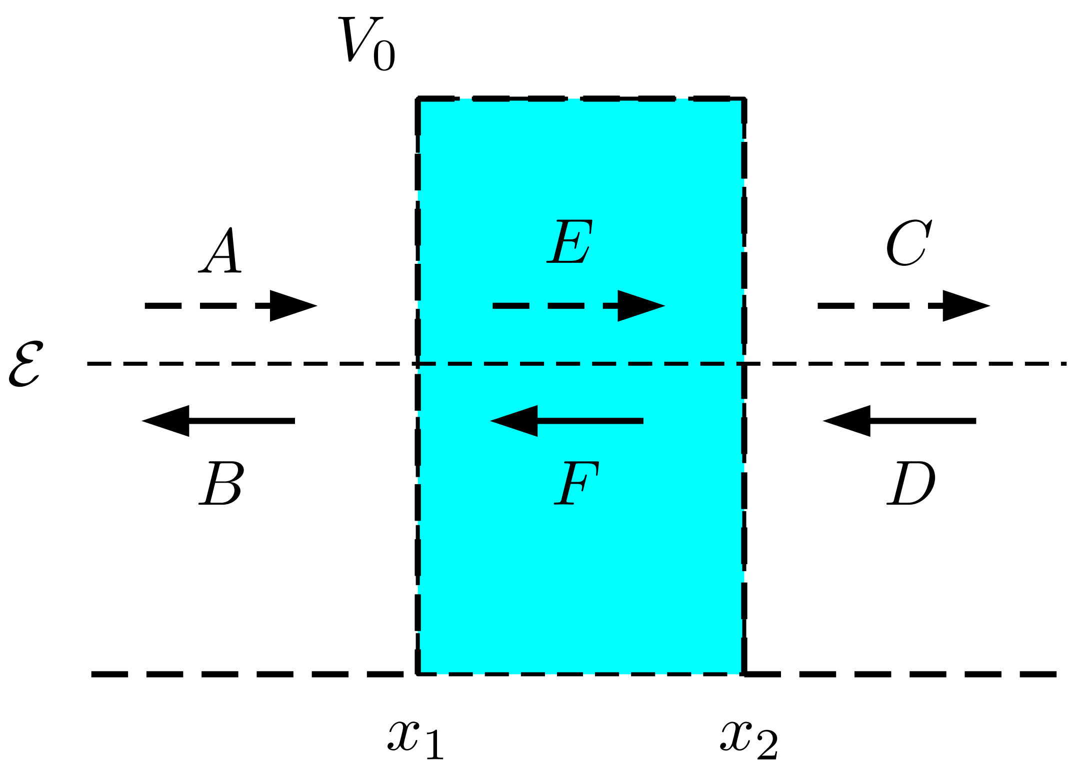

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$$M_4^{-1}M_3M_2^{-1}M_1\begin{bmatrix}A\\B\end{bmatrix}=\begin{bmatrix}C\\D\end{bmatrix}$$

$$\mathcal{M}=M_4^{-1}M_3M_2^{-1}M_1,\qquad\begin{bmatrix}C\\D\end{bmatrix}=\mathcal{M}\begin{bmatrix}A\\B\end{bmatrix}$$

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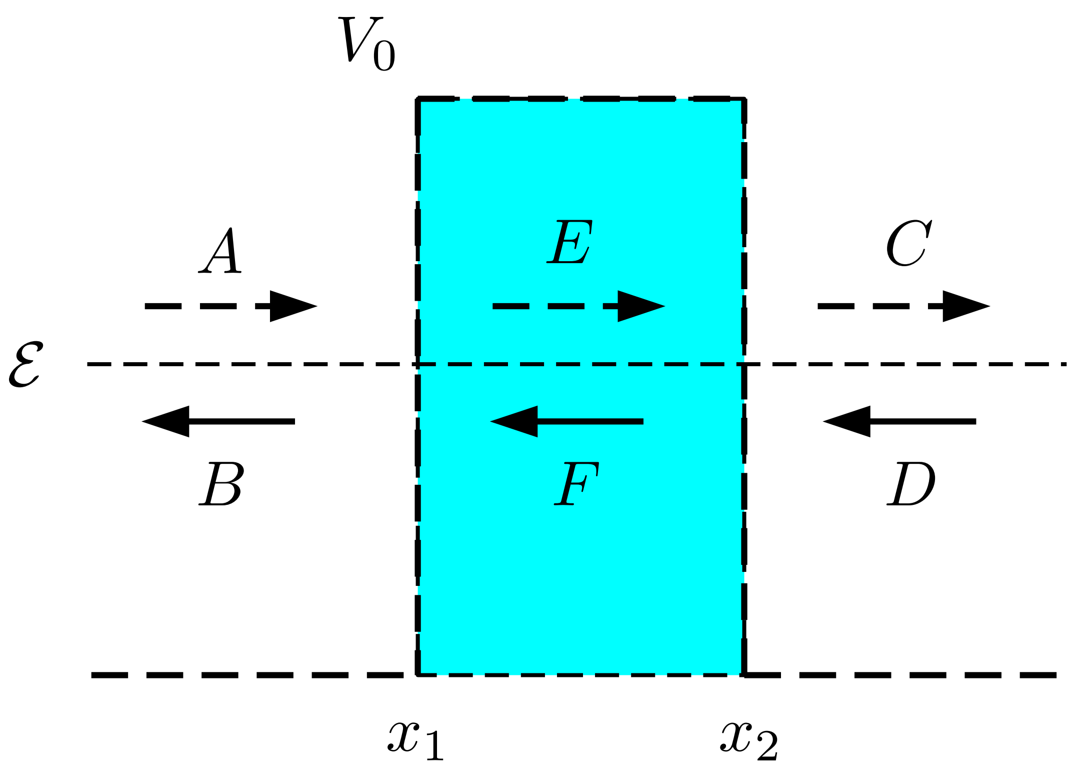

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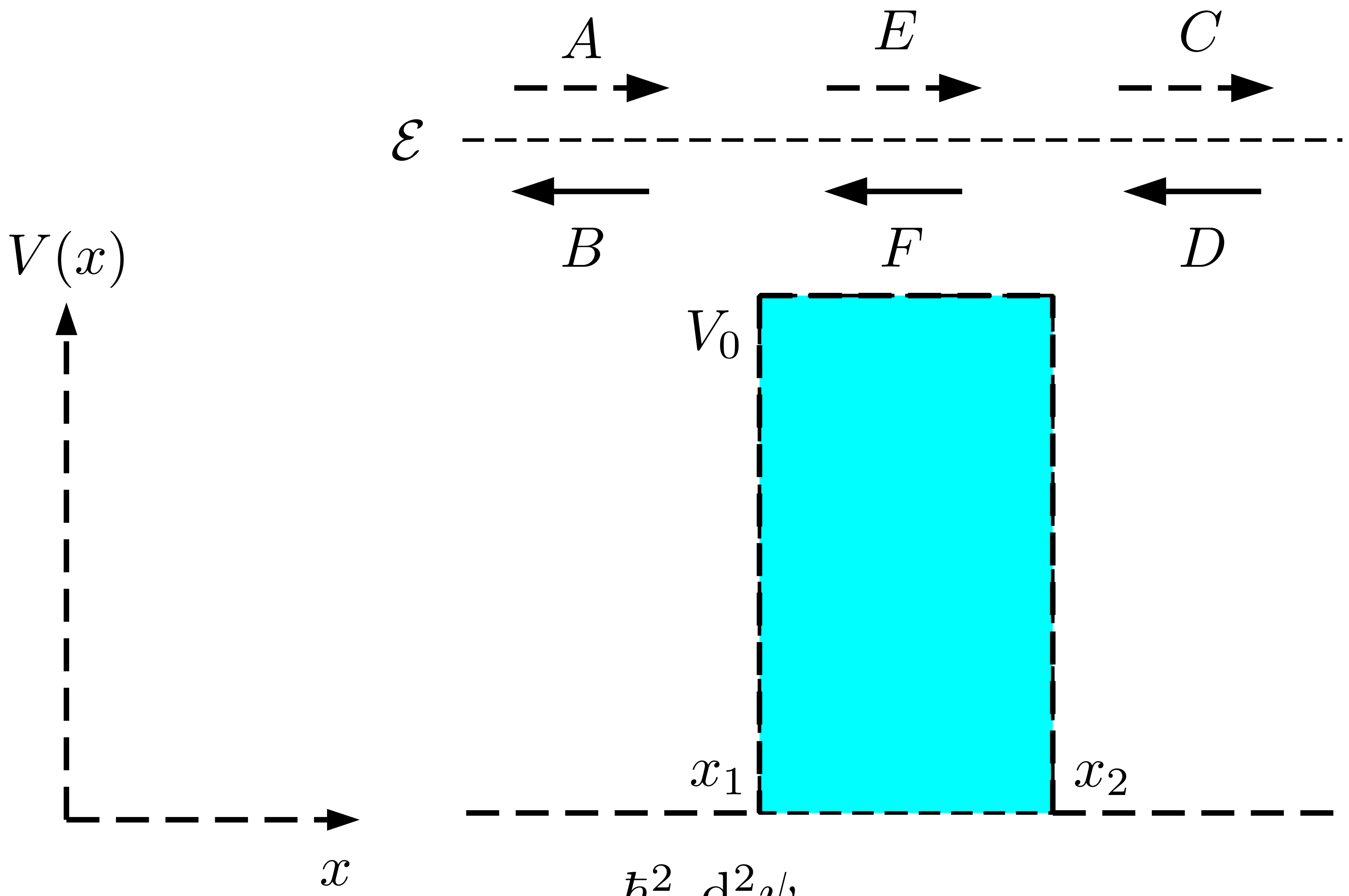

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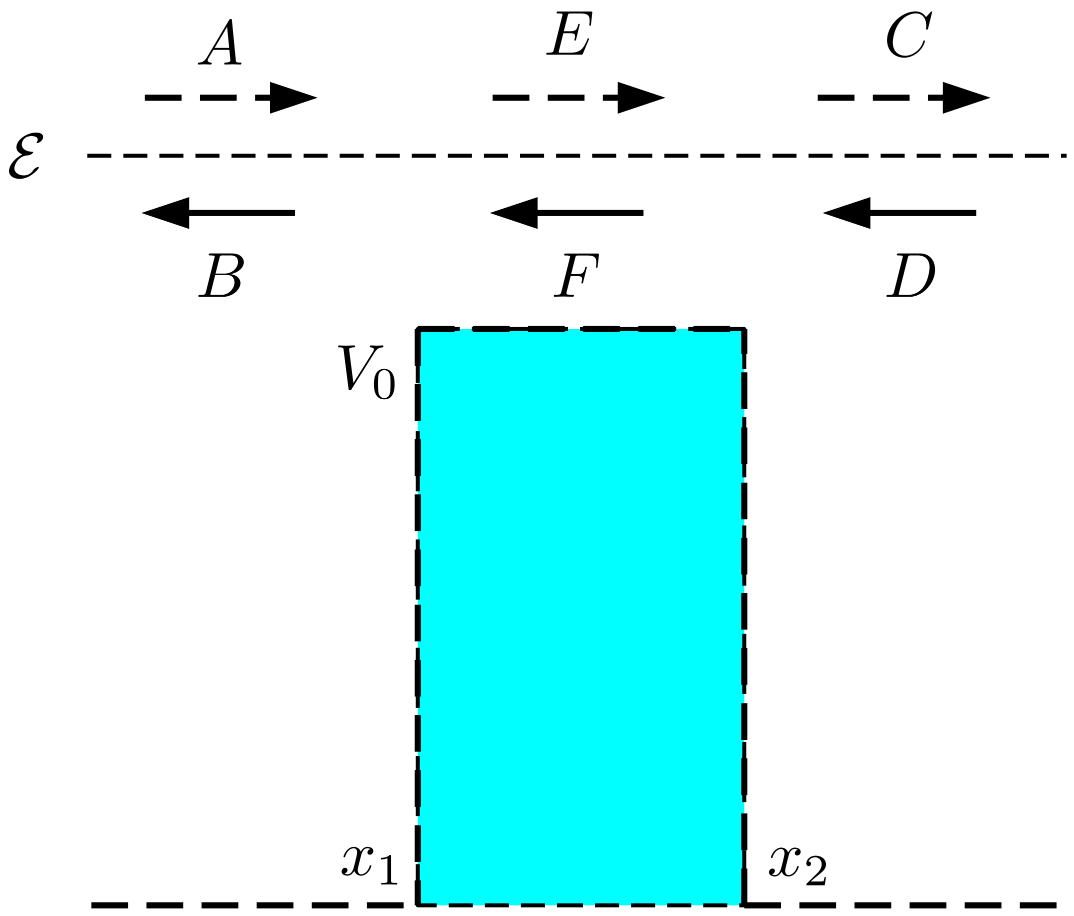

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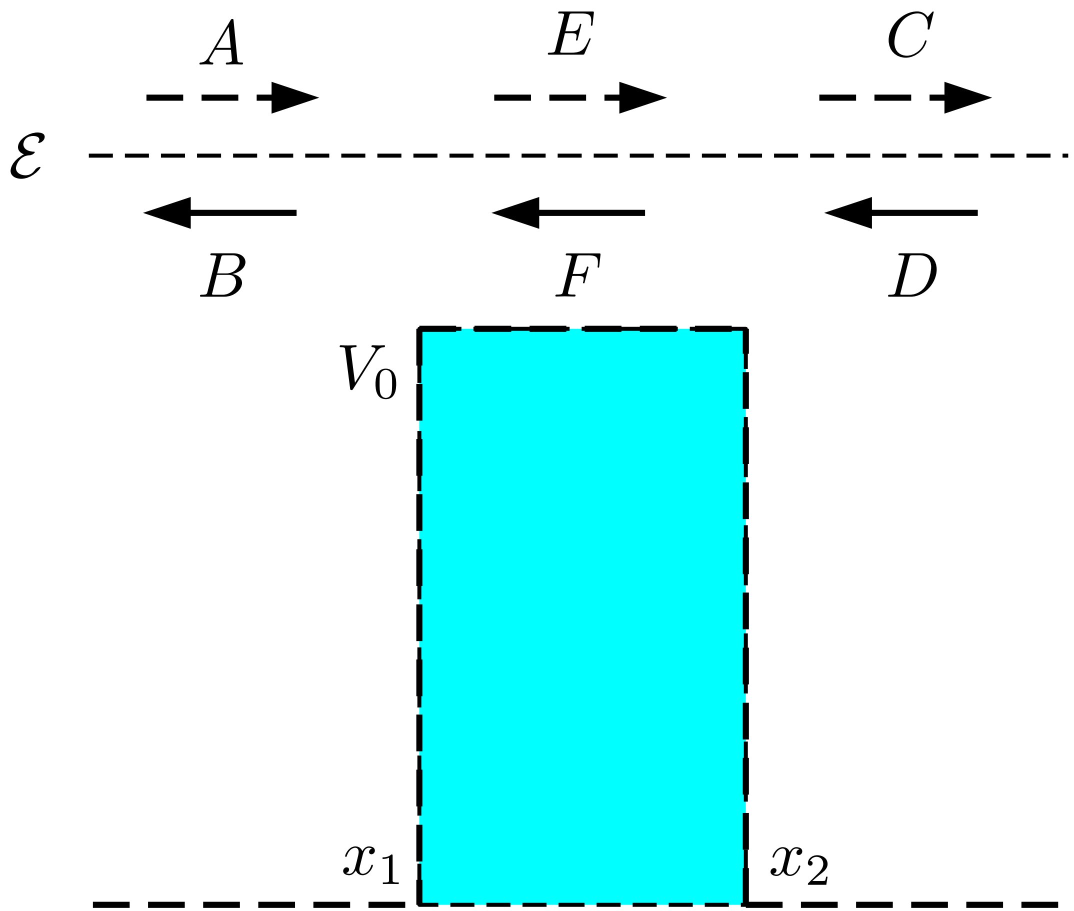

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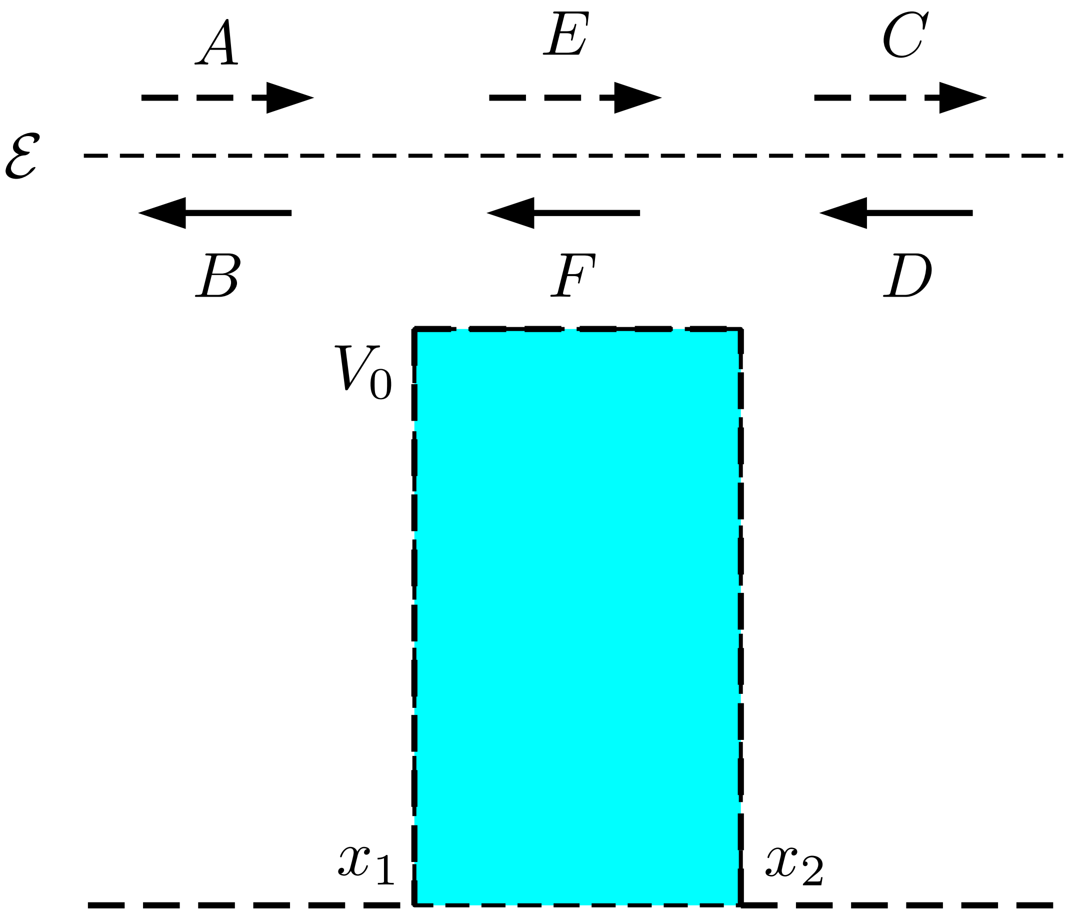

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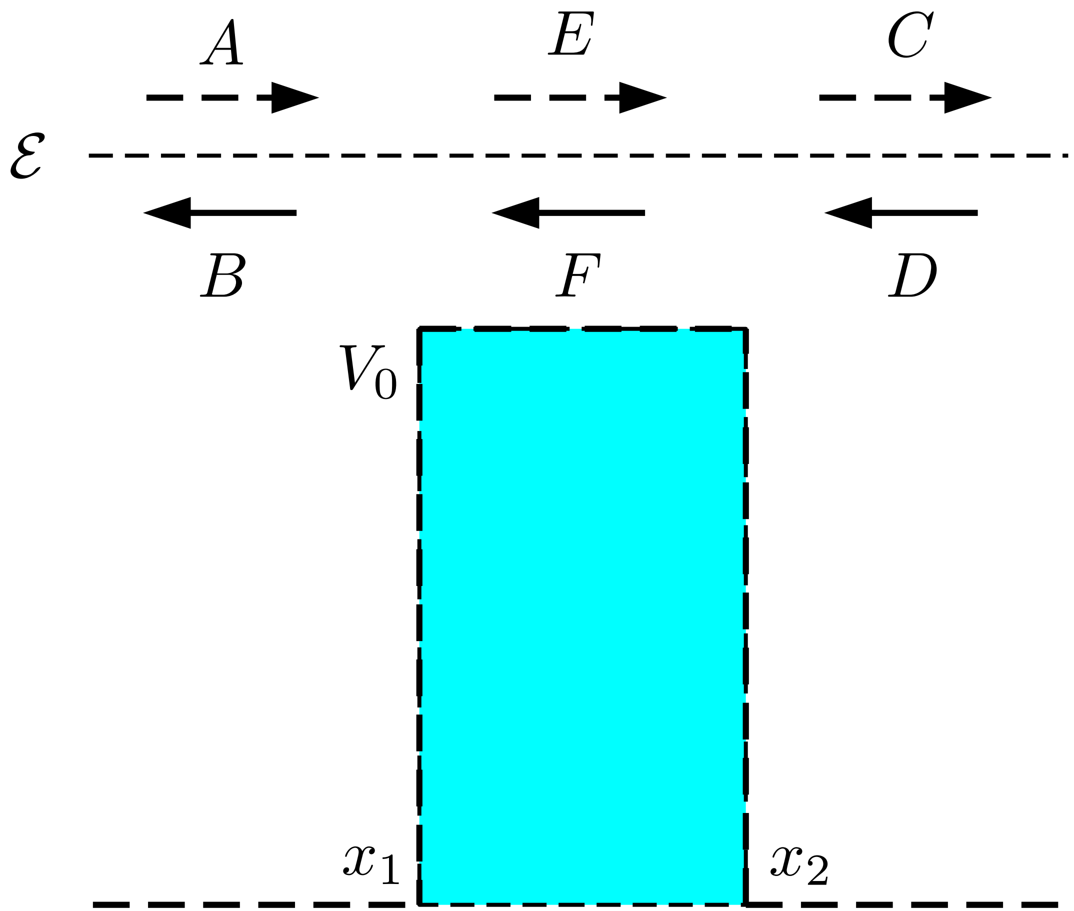
Transmission of Rectangular Barriers



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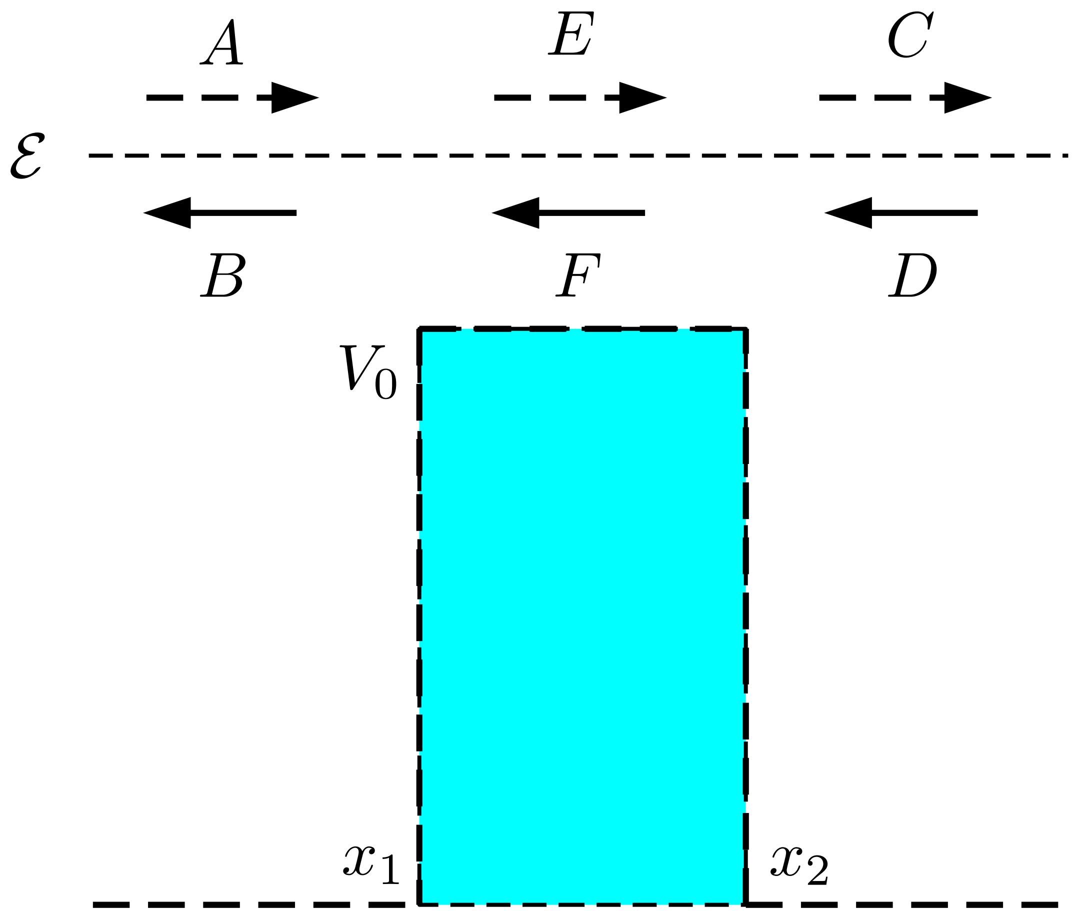
Transmission of Rectangular Barriers



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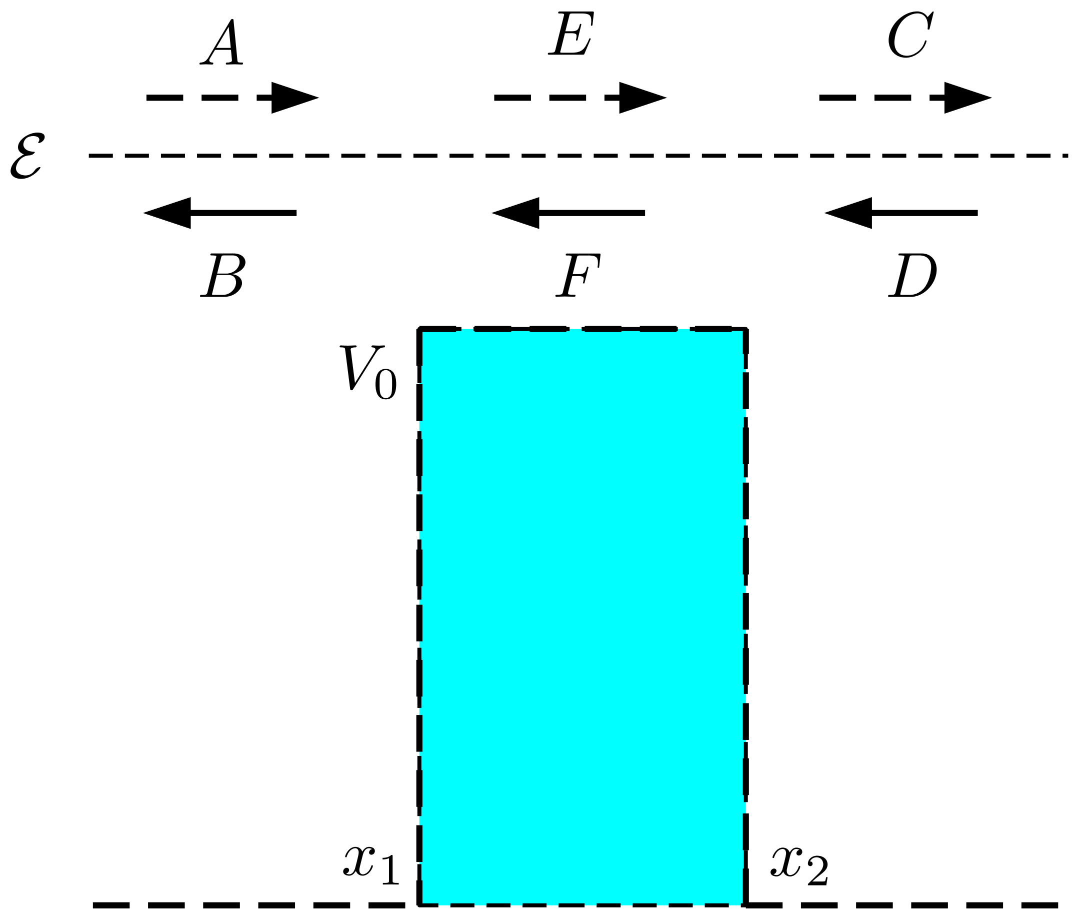

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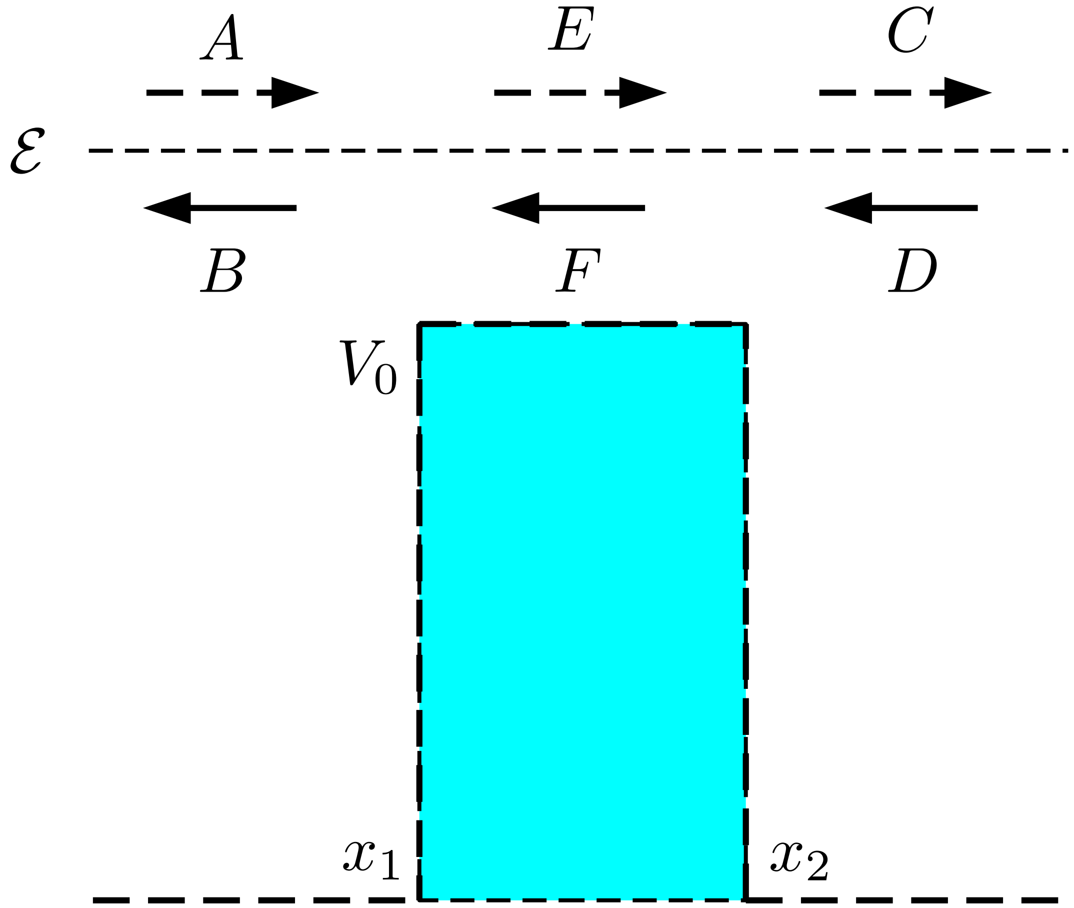
Transmission of Rectangular Barriers



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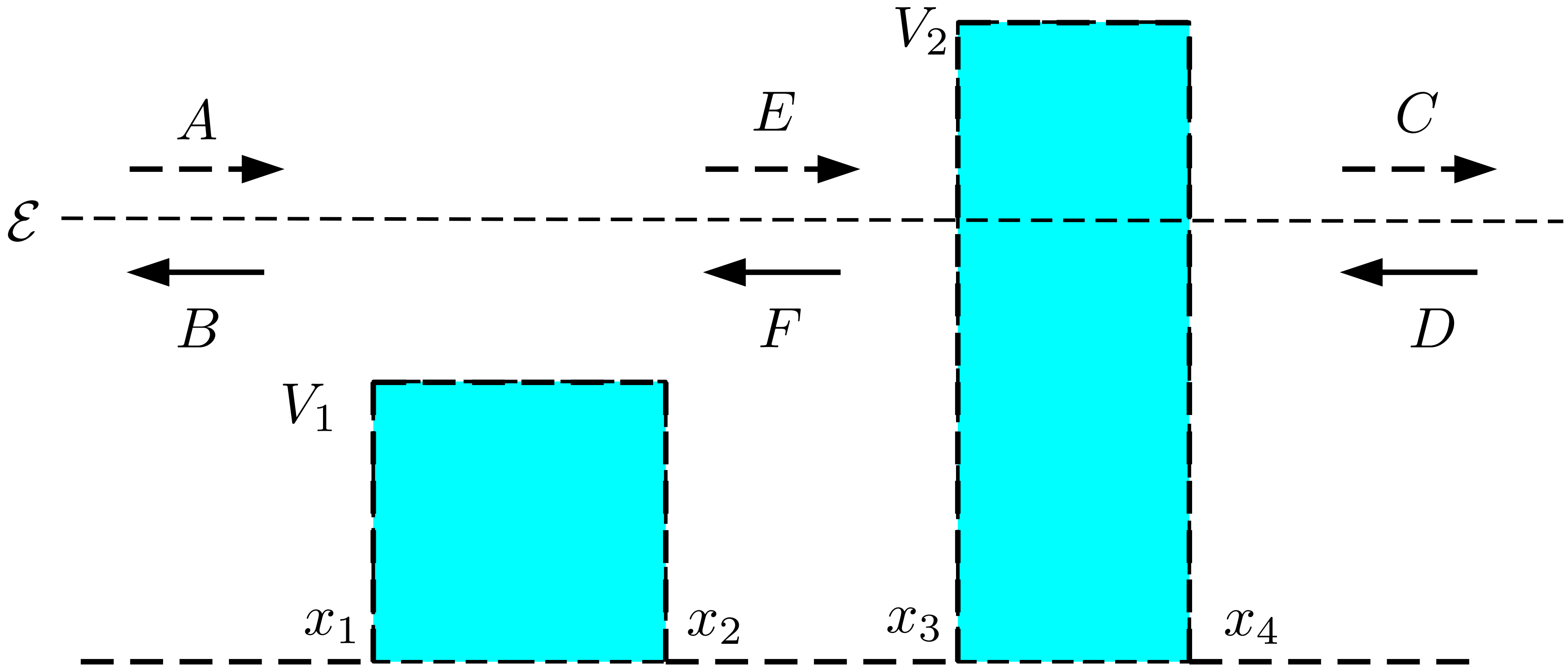
$$\mathcal{M} = M_4^{-1}M_3M_2^{-1}M_1, \qquad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M}\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\mathcal{M} = \mathcal{M}(V_0, \mathcal{E}, x_1, x_2)$$

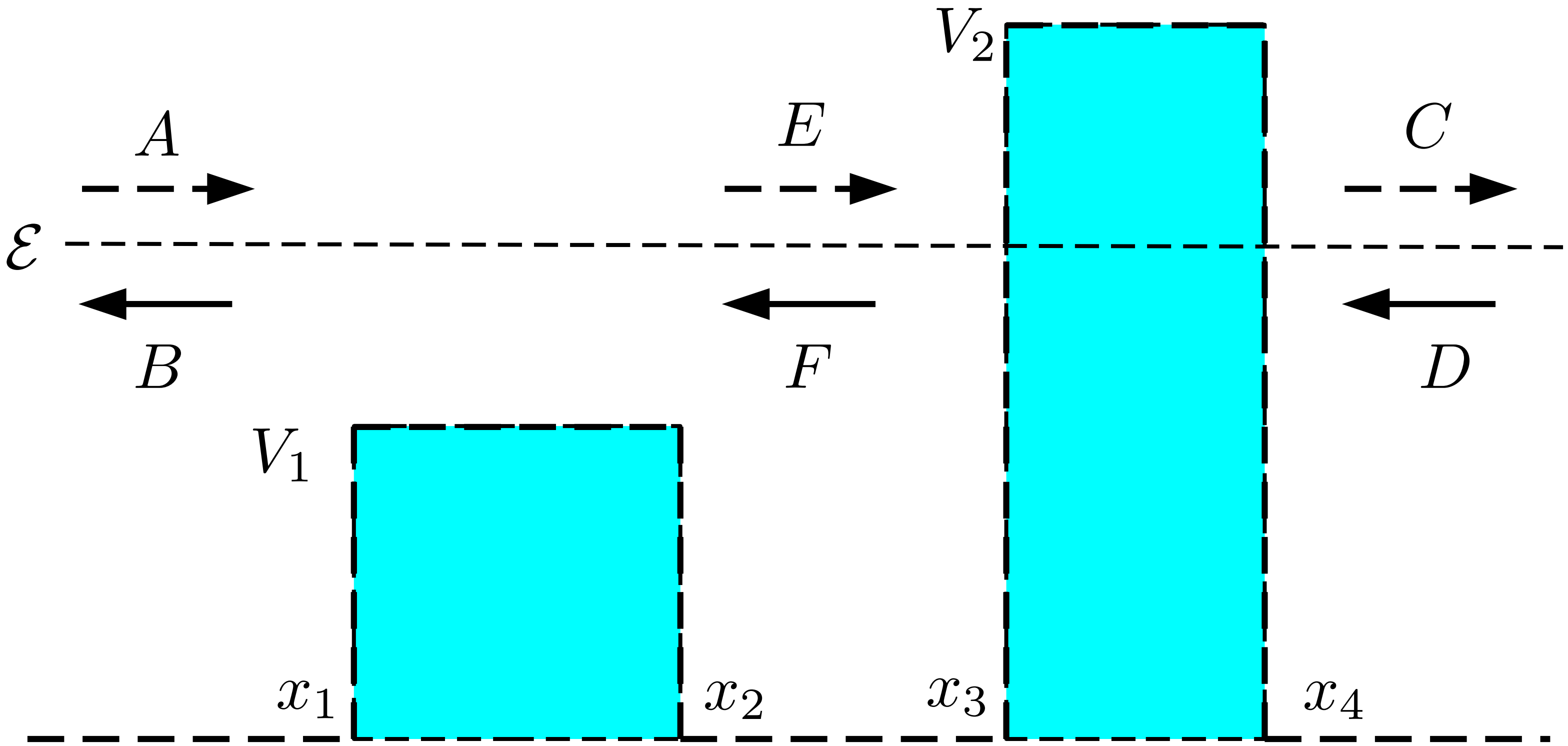
$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(\mathcal{E} - V_0)}{\hbar^2}}$$

Transmission of Rectangular Barriers



$$\begin{bmatrix} E \\ F \end{bmatrix} = \mathcal{M}_1 \begin{bmatrix} A \\ B \end{bmatrix} \qquad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M}_2 \begin{bmatrix} E \\ F \end{bmatrix}$$

Transmission of Rectangular Barriers



$$\mathcal{M}_1 = \mathcal{M}_1(V_1, \mathcal{E}, x_1, x_2)$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(\mathcal{E} - V_1)}{\hbar^2}}$$

$$\mathcal{M}_2 = \mathcal{M}_2(V_2, \mathcal{E}, x_3, x_4)$$

$$k = \sqrt{\frac{2m\mathcal{E}}{\hbar^2}}, \quad q = \sqrt{\frac{2m(V_2 - \mathcal{E})}{\hbar^2}}$$

$$\begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M}_2\mathcal{M}_1 \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} E \\ F \end{bmatrix} = \mathcal{M}_1 \begin{bmatrix} A \\ B \end{bmatrix} \qquad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M}_2 \begin{bmatrix} E \\ F \end{bmatrix}$$

$$\mathcal{M} = \mathcal{M}_2\mathcal{M}_1, \quad \begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}$$

Transmission of Rectangular Barriers

$$\begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}, \quad D = 0$$

$$\begin{bmatrix} C \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\mathcal{M}_{21} A + \mathcal{M}_{22} B = 0 \Rightarrow \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$C = \mathcal{M}_{11} A + \mathcal{M}_{12} B \Rightarrow \frac{C}{A} = \mathcal{M}_{11} + \mathcal{M}_{12} \frac{B}{A}$$

$$r = \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$t = \frac{C}{A} = \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$R = |r|^2 = rr^* = \left| \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}} \right|^2$$

$$T = |t|^2 = tt^* = \left| \mathcal{M}_{11} - \mathcal{M}_{12} \frac{\mathcal{M}_{21}}{\mathcal{M}_{22}} \right|^2$$

Transmission of Rectangular Barriers

$$\begin{bmatrix} C \\ D \end{bmatrix} = \mathcal{M} \begin{bmatrix} A \\ B \end{bmatrix}, \quad D = 0$$

$$\begin{bmatrix} C \\ 0 \end{bmatrix} = \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$r = \frac{B}{A} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$t = \frac{C}{A} = \mathcal{M}_{11} - \mathcal{M}_{12}\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}$$

$$R = |r|^2 = rr^* = \left|\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}\right|^2$$

$$T = |t|^2 = tt^* = \left|\mathcal{M}_{11} - \mathcal{M}_{12}\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}\right|^2$$

$$T(\mathcal{E}) + R(\mathcal{E}) = 1$$

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