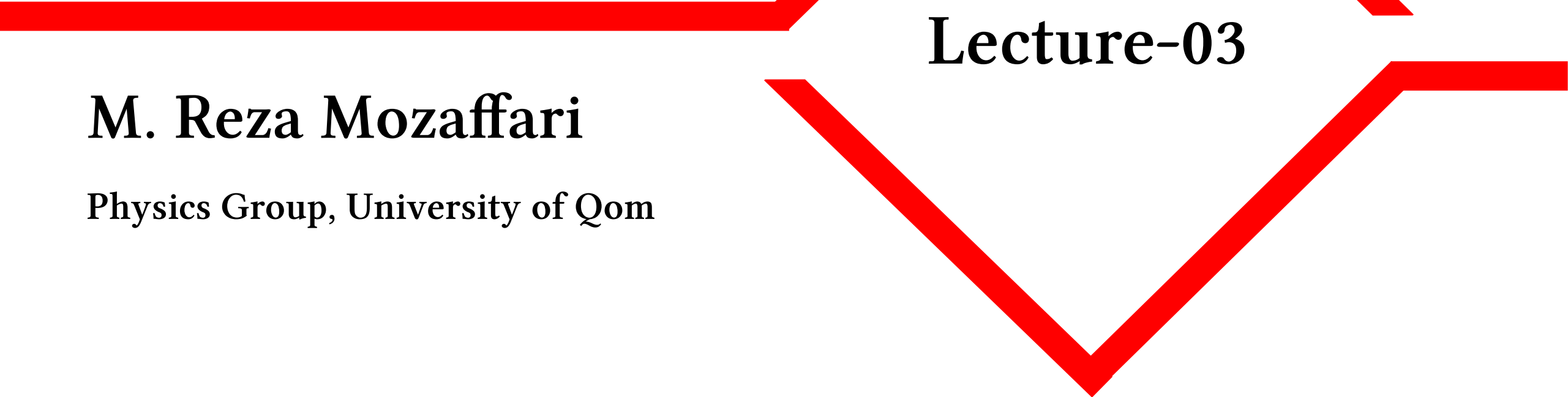


Computational Physics



Lecture-25

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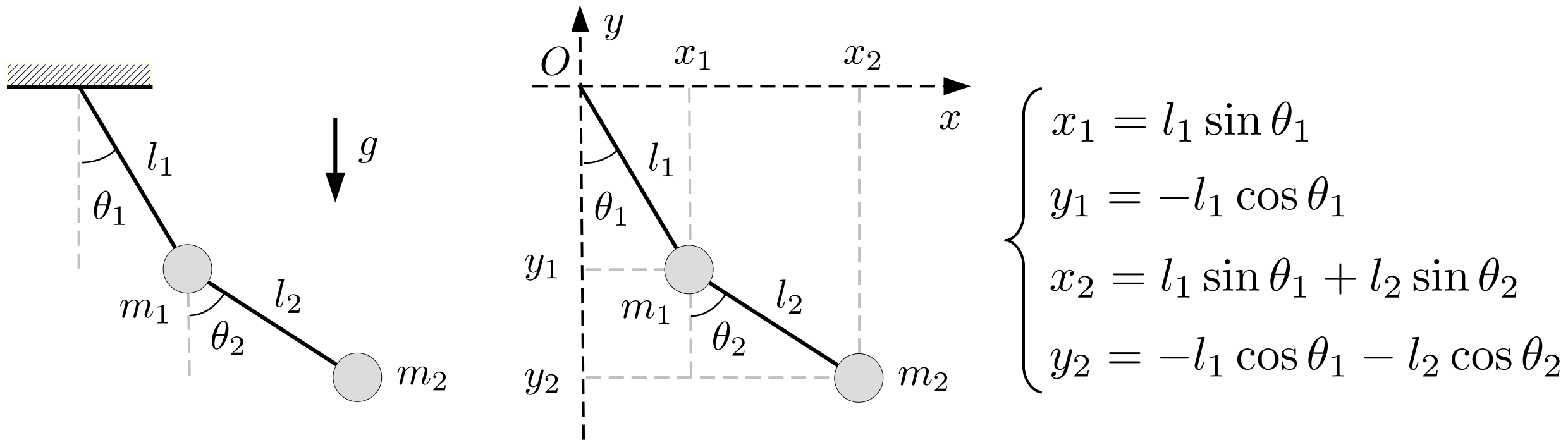
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Contents

- Basis Concepts
- Numerical Differentiation
- Numerical Integration
- Numerical Finding Root
- Classical Scattering
- Solving Linear Systems
- Transmission of Rectangular Barriers
- Approximation of a Function
- Ordinary Differential Equation (Initial Value Problem)
- Double Pendulum

Ordinary Differential Equation

- Double Pendulum



$$\mathcal{L} = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

Ordinary Differential Equation

- Double Pendulum

$$\begin{cases} x_1 = l_1 \sin \theta_1 \\ y_1 = -l_1 \cos \theta_1 \\ x_2 = l_1 \sin \theta_1 + l_2 \sin \theta_2 \\ y_2 = -l_1 \cos \theta_1 - l_2 \cos \theta_2 \end{cases} \quad \begin{cases} \dot{x}_1 = l_1 \dot{\theta}_1 \cos \theta_1 \\ \dot{y}_1 = l_1 \dot{\theta}_1 \sin \theta_1 \\ \dot{x}_2 = l_1 \dot{\theta}_1 \cos \theta_1 + l_2 \dot{\theta}_2 \cos \theta_2 \\ \dot{y}_2 = l_1 \dot{\theta}_1 \sin \theta_1 + l_2 \dot{\theta}_2 \sin \theta_2 \end{cases}$$

$$\mathcal{L} = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)) \\ + m_1 g l_1 \cos \theta_1 + m_2 g (l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$\frac{d}{dt} [(m_1 + m_2)l_1^2\dot{\theta}_1 + m_2l_1l_2\dot{\theta}_2 \cos(\theta_1 - \theta_2)]$$
$$= -m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)gl_1 \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$(m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_2(\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$= -m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)gl_1 \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$(m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_2(\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$

$$= -m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)gl_1 \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$(m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2)$$
$$= -(m_1 + m_2)gl_1 \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$(m_1 + m_2)l_1^2\ddot{\theta}_1 + m_2l_1l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2)$$
$$= -(m_1 + m_2)gl_1 \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = \frac{\partial \mathcal{L}}{\partial \theta_1}$$

$$(m_1 + m_2)l_1\ddot{\theta}_1 + m_2l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) = -m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)g \sin \theta_1$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$\frac{d}{dt} [m_2l_2^2\dot{\theta}_2 + m_2l_1l_2\dot{\theta}_1 \cos(\theta_1 - \theta_2)]$$

$$= m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2gl_2 \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$m_2l_2^2\ddot{\theta}_2 + m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$
$$= m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2gl_2 \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$m_2l_2^2\ddot{\theta}_2 + m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_1(\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)$$

$$= m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \sin(\theta_1 - \theta_2) - m_2gl_2 \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$m_2l_2^2\ddot{\theta}_2 + m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) = -m_2gl_2 \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$m_2l_2^2\ddot{\theta}_2 + m_2l_1l_2\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1l_2\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) = -m_2gl_2 \sin \theta_2$$

$$m_2l_2\ddot{\theta}_2 + m_2l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) = -m_2g \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$\cancel{m_2}l_2\ddot{\theta}_2 + \cancel{m_2}l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \cancel{m_2}l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) = -\cancel{m_2}g \sin \theta_2$$

$$l_2\ddot{\theta}_2 + l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) = -g \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$
$$+ m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = \frac{\partial \mathcal{L}}{\partial \theta_2}$$

$$l_2\ddot{\theta}_2 + l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) = l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - g \sin \theta_2$$

Ordinary Differential Equation

- Double Pendulum

$$\mathcal{L} = \frac{1}{2}m_1l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2))$$

$$+m_1gl_1 \cos \theta_1 + m_2g(l_1 \cos \theta_1 + l_2 \cos \theta_2)$$

$$\begin{cases} (m_1 + m_2)l_1\ddot{\theta}_1 + m_2l_2\ddot{\theta}_2 \cos(\theta_1 - \theta_2) = -m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)g \sin \theta_1 \\ l_1\ddot{\theta}_1 \cos(\theta_1 - \theta_2) + l_2\ddot{\theta}_2 = l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - g \sin \theta_2 \end{cases}$$

$$\begin{bmatrix} (m_1 + m_2)l_1 & m_2l_2 \cos(\theta_1 - \theta_2) \\ l_1 \cos(\theta_1 - \theta_2) & l_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)g \sin \theta_1 \\ l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - g \sin \theta_2 \end{bmatrix}$$

Ordinary Differential Equation

- Double Pendulum

$$\begin{bmatrix} (m_1 + m_2)l_1 & m_2l_2 \cos(\theta_1 - \theta_2) \\ l_1 \cos(\theta_1 - \theta_2) & l_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)g \sin \theta_1 \\ l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - g \sin \theta_2 \end{bmatrix}$$

$$\ddot{\theta}_1 = \frac{-(2m_1 + m_2)g \sin \theta_1 - m_2g \sin(\theta_1 - 2\theta_2) - 2m_2 \sin(\theta_1 - \theta_2)[l_2\dot{\theta}_2^2 + l_1\dot{\theta}_1^2 \cos(\theta_1 - \theta_2)]}{l_1(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

$$\ddot{\theta}_2 = \frac{2 \sin(\theta_1 - \theta_2)[(m_1 + m_2)l_1g \cos \theta_1 + (m_1 + m_2)l_1\dot{\theta}_1^2 + m_2l_2\dot{\theta}_2^2 \cos(\theta_1 - \theta_2)]}{l_2(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

Ordinary Differential Equation

- Double Pendulum

$$\begin{bmatrix} (m_1 + m_2)l_1 & m_2l_2 \cos(\theta_1 - \theta_2) \\ l_1 \cos(\theta_1 - \theta_2) & l_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -m_2l_2\dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - (m_1 + m_2)g \sin \theta_1 \\ l_1\dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - g \sin \theta_2 \end{bmatrix}$$

$$\dot{\theta}_1 = \omega_1$$

$$\dot{\theta}_2 = \omega_2$$

$$\dot{\omega}_1 = \frac{-(2m_1 + m_2)g \sin \theta_1 - m_2g \sin(\theta_1 - 2\theta_2) - 2m_2 \sin(\theta_1 - \theta_2)[l_2\omega_2^2 + l_1\omega_1^2 \cos(\theta_1 - \theta_2)]}{l_1(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

$$\dot{\omega}_2 = \frac{2 \sin(\theta_1 - \theta_2)[(m_1 + m_2)l_1g \cos \theta_1 + (m_1 + m_2)l_1\omega_1^2 + m_2l_2\omega_2^2 \cos(\theta_1 - \theta_2)]}{l_2(2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$