جلسه هفدهم

مکانیک آماری

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اسفند ۹۸

فوتون‌ها

$$\omega = ck$$

$$g_{3D}(E) = (2S+1)\sum_k \delta(E - E_k)$$

$S = 1 \Rightarrow \; 2S+1 \neq 3$

قطبش راستگرد

قطبش چپگرد

$$g_{3D}(E) = 2\sum_k \delta(E - E_k)$$

$$\begin{cases} g_{3D}(E) = 2\sum_k \delta(E - \hbar ck) \\ \sum_k \rightarrow \dfrac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 \mathrm{d}k \end{cases} \Longrightarrow g_{3D}(E) = 2\frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 \mathrm{d}k\, \delta(E - \hbar ck)$$

فوتون‌ها

$$g_{3D}(E) = 2\frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 \mathrm{d}k\,\delta(E-\hbar ck)$$

$$\delta(E-\hbar ck) = \frac{\delta(k-k_0)}{\hbar c}, \quad k_0 = \frac{E}{\hbar c}$$

$$g_{3D}(E) = 2\frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 \mathrm{d}k\frac{\delta(k-k_0)}{\hbar c} = \frac{V}{\pi^2}\frac{k_0^2}{\hbar c} = \frac{VE^2}{\pi^2\hbar^3c^3}$$

$$\boxed{g_{3D}(E) = \frac{VE^2}{\pi^2\hbar^3c^3}}$$

$$N = \int_0^\infty \frac{g_{3D}(E)\mathrm{d}E}{e^{\beta E-\mu}-1}, \quad ,U = \int_0^\infty \frac{g_{3D}(E)E\mathrm{d}E}{e^{\beta E}-1}$$

فوتون‌ها

$$\boxed{g_{3D}(E) = \frac{VE^2}{\pi^2\hbar^3c^3}}$$

$$\mu = 0$$

$$N = \int_0^\infty \frac{g_{3D}(E)\mathrm{d}E}{e^{\beta E}-1}, \quad U = \int_0^\infty \frac{g_{3D}(E)E\mathrm{d}E}{e^{\beta E}-1}$$

$$N = \frac{V}{\pi^2\hbar^3c^3}\int_0^\infty \frac{E^2\mathrm{d}E}{e^{\beta E}-1}, \quad \beta E = x \Rightarrow \mathrm{d}E = k_BT\mathrm{d}x$$

$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^2\mathrm{d}x}{e^x-1}$$

فوتون‌ها

$$\boxed{g_{3D}(E) = \frac{VE^2}{\pi^2\hbar^3 c^3}}$$

$$\mu = 0$$

$$N = \frac{V(k_B T)^3}{\pi^2\hbar^3 c^3}\int_0^\infty \frac{x^2 \mathrm{d}x}{e^x - 1}, \quad U = \int_0^\infty \frac{g_{3D}(E)E\mathrm{d}E}{e^{\beta E} - 1}$$

$$U = \frac{V}{\pi^2\hbar^3 c^3}\int_0^\infty \frac{E^3 \mathrm{d}E}{e^{\beta E} - 1}, \quad \beta E = x \Rightarrow \mathrm{d}E = k_B T \mathrm{d}x$$

$$U = \frac{V(k_B T)^4}{\pi^2\hbar^3 c^3}\int_0^\infty \frac{x^3 \mathrm{d}x}{e^x - 1}$$

فوتون‌ها

$$g_{3D}(E) = \frac{VE^2}{\pi^2\hbar^3c^3}$$

$$\mu = 0$$

$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^2\mathrm{d}x}{e^x-1}, \quad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^3\mathrm{d}x}{e^x-1}$$

$$\int_0^\infty \frac{x^2\mathrm{d}x}{e^x-1} = ?, \quad \int_0^\infty \frac{x^3\mathrm{d}x}{e^x-1} = ?$$

انتگرال بوز: $I_B(n) = \int_0^\infty \frac{x^n\mathrm{d}x}{e^x-1}$

فوتون‌ها

$$N = \frac{V(k_B T)^3}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^2 \mathrm{d}x}{e^x - 1}, \qquad U = \frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^3 \mathrm{d}x}{e^x - 1}$$

انتگرال بوز : $I_B(n) = \int_0^\infty \frac{x^n \mathrm{d}x}{e^x - 1}$

$$I_B(n) = \int_0^\infty \frac{x^n \mathrm{d}x}{e^x - 1} = \int_0^\infty \frac{x^n e^{-x} \mathrm{d}x}{1 - e^{-x}} = \int_0^\infty \left(\sum_{k=0}^\infty e^{-kx} \right) x^n e^{-x} \mathrm{d}x$$

$$= \sum_{k=0}^\infty \int_0^\infty x^n e^{-(k+1)x} \mathrm{d}x, \qquad (k+1)x = y \Rightarrow \mathrm{d}x = \frac{1}{k+1} \mathrm{d}y$$

$$= \sum_{k=0}^\infty \frac{1}{(k+1)^{n+1}} \int_0^\infty y^n e^{-y} \mathrm{d}y$$

فوتون‌ها

$$N = \frac{V(k_B T)^3}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^2 \mathrm{d}x}{e^x - 1}, \qquad U = \frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^3 \mathrm{d}x}{e^x - 1}$$

انتگرال بوز : $$I_B(n) = \int_0^\infty \frac{x^n \mathrm{d}x}{e^x - 1}$$

$$I_B(n) = \sum_{k=0}^\infty \frac{1}{(k+1)^{n+1}} \int_0^\infty y^n e^{-y} \mathrm{d}y$$

تعریف تابع گاما : $$\int_0^\infty y^n e^{-y} \mathrm{d}y = \Gamma(n+1)$$

$$I_B(n) = \sum_{k=0}^\infty \frac{1}{(k+1)^{n+1}} \Gamma(n+1) = \Gamma(n+1) \sum_{k=0}^\infty \frac{1}{(k+1)^{n+1}}$$

فوتون‌ها

$$N = \frac{V(k_B T)^3}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^2 \mathrm{d}x}{e^x - 1}, \quad U = \frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^3 \mathrm{d}x}{e^x - 1}$$

انتگرال بوز : $I_B(n) = \int_0^\infty \frac{x^n \mathrm{d}x}{e^x - 1}$

$$I_B(n) = \Gamma(n+1) \sum_{k=0}^{\infty} \frac{1}{(k+1)^{n+1}}$$

(Zeta) تعریف زتا : $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$

$$\sum_{k=0}^{\infty} \frac{1}{(k+1)^{n+1}}, \quad \boxed{\begin{array}{c} k+1 = m \\ k=0 \Rightarrow m=1,\ k \to \infty \Rightarrow m \to \infty \end{array}} \quad \sum_{m=1}^{\infty} \frac{1}{m^{n+1}} = \zeta(n+1)$$

فوتون‌ها

$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^2\mathrm{d}x}{e^x-1}, \qquad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^3\mathrm{d}x}{e^x-1}$$

انتگرال بوز : $I_B(n) = \int_0^\infty \frac{x^n\mathrm{d}x}{e^x-1}$

$$I_B(n) = \Gamma(n+1)\sum_{k=0}^{\infty}\frac{1}{(k+1)^{n+1}}$$

$$\sum_{k=0}^{\infty}\frac{1}{(k+1)^{n+1}} = \zeta(n+1)$$

$$I_B(n) = \Gamma(n+1)\zeta(n+1)$$

فوتون‌ها

$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^2\mathrm{d}x}{e^x-1}, \quad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}\int_0^\infty \frac{x^3\mathrm{d}x}{e^x-1}$$

انتگرال بوز : $I_B(n) = \int_0^\infty \frac{x^n\mathrm{d}x}{e^x-1} = \Gamma(n+1)\zeta(n+1)$

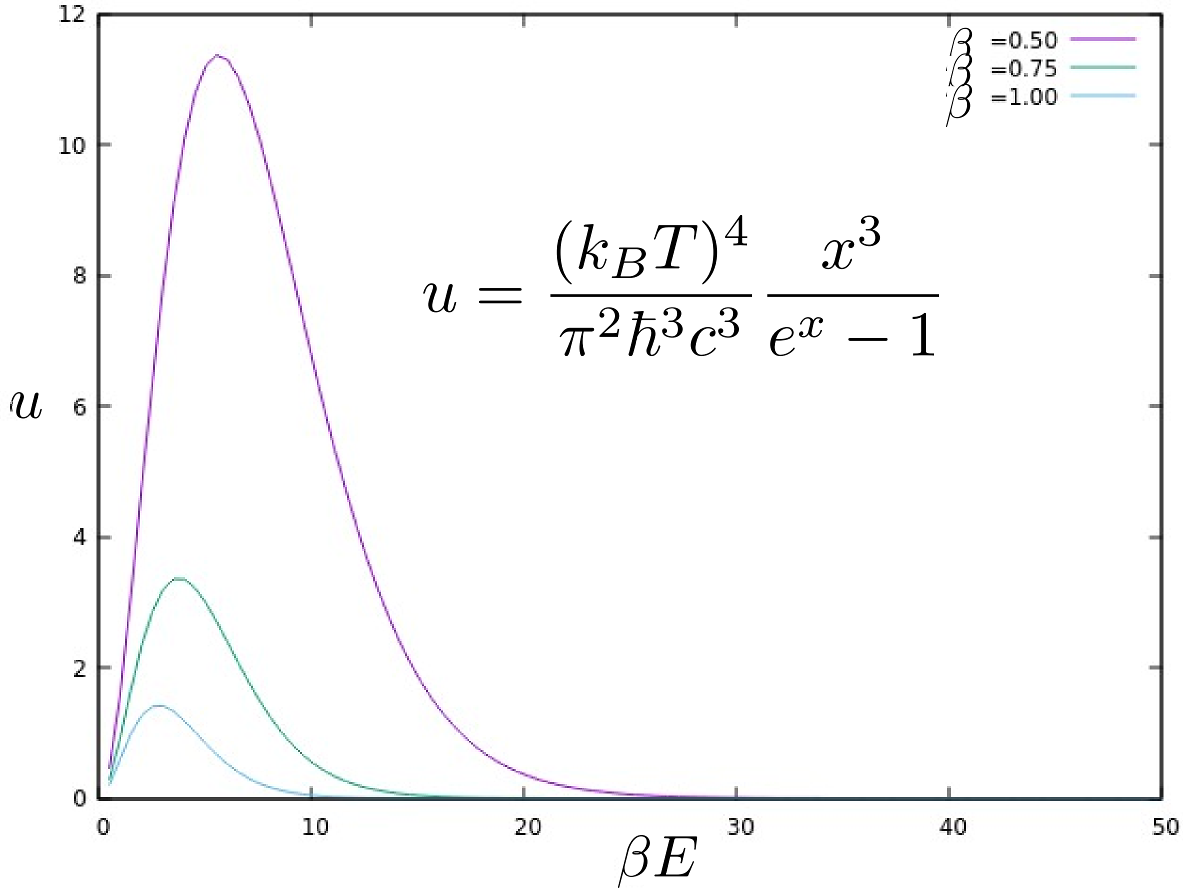
$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\Gamma(3)\zeta(3), \quad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}\Gamma(4)\zeta(4)$$

$$N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}2!\zeta(3), \quad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}3! \times \frac{\pi^4}{90}$$

$$\boxed{N = \frac{V(k_BT)^3}{\pi^2\hbar^3c^3}\zeta(3), \quad U = \frac{V(k_BT)^4}{\pi^2\hbar^3c^3}\frac{\pi^4}{15}}$$

فوتون‌ها

$$U = \frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^3 \mathrm{d}x}{e^x - 1} = \frac{V \pi^2 k_B^4}{15 \hbar^3 c^3} T^4$$



تابش جسم سیاه

$$u = \frac{U}{V} = \frac{4\sigma}{c} T^4$$

ثابت استفان-بولتزمن

$$\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}$$

$$\sigma = 5.67 \times 10^{-8} \mathrm{W\,m^{-2}\,K^{-4}}$$

فوتون‌ها

$$\mu = 0$$

$$\Phi_{\rm G} = k_B T \int \mathrm{d}E\, g_{3D}(E) \ln[1 - e^{-\beta E}]$$

$$g_{3D}(E) = \frac{V E^2}{\pi^2 \hbar^3 c^3}$$

$$\Phi_{\rm G} = \frac{V k_B T}{\pi^2 \hbar^3 c^3} \int_0^\infty E^2 \ln[1 - e^{-\beta E}]\mathrm{d}E, \quad \beta E = x \Rightarrow \mathrm{d}E = k_B T \mathrm{d}x$$

$$\Phi_{\rm G} = \frac{V (k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty x^2 \ln[1 - e^{-x}]\mathrm{d}x$$

$$\frac{1}{1-u} = \sum_{n=0}^{\infty} u^n, \qquad \int \frac{\mathrm{d}u}{1-u} = \sum_{n=0}^{\infty} \int u^n \mathrm{d}u, \qquad -\ln(1-u) = \sum_{n=0}^{\infty} \frac{1}{n+1} u^{n+1}$$

فوتون‌ها

$$-\ln(1-u) = \sum_{n=0}^{\infty} \frac{1}{n+1} u^{n+1}$$

$$\Phi_{\mathrm{G}} = \frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty x^2 \ln[1 - e^{-x}]\mathrm{d}x = -\frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \sum_{n=0}^{\infty} \frac{1}{n+1} \int_0^\infty x^2 e^{-(n+1)x}\mathrm{d}x$$

$$(n+1)x = v \Rightarrow \mathrm{d}x = \frac{\mathrm{d}v}{n+1}, \qquad \Phi_{\mathrm{G}} = -\frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3} \sum_{n=0}^{\infty} \frac{1}{(n+1)^4} \int_0^\infty v^2 e^{-v}\mathrm{d}v$$

$$\int_0^\infty v^2 e^{-v}\mathrm{d}v = \Gamma(3) \qquad \sum_{k=0}^{\infty} \frac{1}{(k+1)^{n+1}} = \zeta(n+1)$$

$$\Phi_{\mathrm{G}} = -\frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3}\zeta(4)\Gamma(3) = -\frac{V(k_B T)^4}{\pi^2 \hbar^3 c^3}\frac{\pi^4}{90} \times 2! = -\frac{V\pi^2(k_B T)^4}{45\hbar^3 c^3}$$

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$$\Phi_{\mathrm{G}} = -\frac{V\pi^2(k_B T)^4}{45\hbar^3 c^3}, \qquad \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}$$

$$\boxed{\Phi_{\mathrm{G}} = -\frac{60\sigma V T^4}{45c} = -\frac{4\sigma V T^4}{3c}}$$

می‌دانیم $\Phi_{\mathrm{G}} = F - \mu N = U - TS - \mu N \xrightarrow{\mu = 0} \Phi_{\mathrm{G}} = F = U - TS$

$$\boxed{F = -\frac{4\sigma V T^4}{3c}}$$

$$-\frac{4\sigma V T^4}{3c} = \frac{4\sigma V T^4}{c} - TS \Rightarrow \boxed{S = \frac{16\sigma V T^4}{3c}}$$

می‌دانیم $\Phi_{\mathrm{G}} = -pV \Rightarrow \boxed{p = \frac{4\sigma T^4}{3c}}$