مکانیک آماری

جلسه بیست و هفتم

محمدرضا مظفری

گروه فیزیک، دانشکده علوم پایه، دانشگاه قم

اسفند ۹۹

آنسامبل میکروکانونیک

اگر آنتروپی $S(N,V,E)$ یک کمیت حجمی باشد، درستی رابطه زیر را بررسی کنید.

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E}+V\left(\frac{\partial S}{\partial V}\right)_{E,N}+E\left(\frac{\partial S}{\partial E}\right)_{N,V}=S$$

اگر انتروپی یک کمیت حجمی باشد،

$$S(N,V,E)=Nf(V/N,E/N)$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E}=N\left[f+N\left(\frac{-V}{N^2}\right)f'_{V/N}+N\left(\frac{-E}{N^2}\right)f'_{E/N}\right]$$

$$V\left(\frac{\partial S}{\partial V}\right)_{E,N}=V\left[N\left(\frac{1}{N}\right)f'_{V/N}\right]$$

$$E\left(\frac{\partial S}{\partial E}\right)_{N,V}=E\left[N\left(\frac{1}{N}\right)f'_{E/N}\right]$$

آنسامبل میکروکانونیک

$$S(N,V,E) = Nf(V/N, E/N)$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} = N\left[f + N\left(\frac{-V}{N^2}\right)f'_{V/N} + N\left(\frac{-E}{N^2}\right)f'_{E/N}\right]$$

$$V\left(\frac{\partial S}{\partial V}\right)_{E,N} = V\left[N\left(\frac{1}{N}\right)f'_{V/N}\right]$$

$$E\left(\frac{\partial S}{\partial E}\right)_{N,V} = E\left[N\left(\frac{1}{N}\right)f'_{E/N}\right]$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} = \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right]$$

$$V\left(\frac{\partial S}{\partial V}\right)_{E,N} = Vf'_{V/N}$$

$$E\left(\frac{\partial S}{\partial E}\right)_{N,V} = Ef'_{E/N}$$

آنسامبل میکروکانونیک

$$S(N,V,E) = Nf(V/N, E/N)$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} = \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right]$$

$$V\left(\frac{\partial S}{\partial V}\right)_{E,N} = Vf'_{V/N}$$

$$E\left(\frac{\partial S}{\partial E}\right)_{N,V} = Ef'_{E/N}$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V} = \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right] + Vf'_{V/N} + Ef'_{E/N}$$

آنسامبل میکروکانونیک

$$S(N,V,E) = Nf(V/N, E/N)$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} = \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right]$$

$$V\left(\frac{\partial S}{\partial V}\right)_{E,N} = Vf'_{V/N}$$

$$E\left(\frac{\partial S}{\partial E}\right)_{N,V} = Ef'_{E/N}$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V}$$
$$= \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right] + Vf'_{V/N} + Ef'_{E/N}$$

آنسامبل میکروکانونیک

$$S(N, V, E) = Nf(V/N, E/N)$$

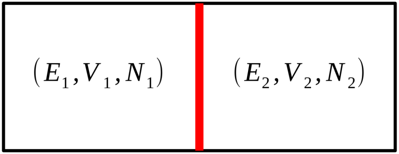
$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V} = \left[Nf - Vf'_{V/N} - Ef'_{E/N}\right] + Vf'_{V/N} + Ef'_{E/N}$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V} = Nf$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V} = S$$

آنسامبل میکروکانونیک

میکروحالت مرکب



$$\Omega(E_1, E_2) = \Omega_1(E_1)\Omega_2(E_2)$$

$$E = E_1 + E_2 \Rightarrow E_2 = E - E_1$$

$$\Omega(E, E_1) = \Omega_1(E_1)\Omega_2(E - E_1)$$

حالت تعادلی سیستم در E_1 وقتی اتفاق می‌افتد که بتواند مقدار میکروحالت مرکب را به حداکثر برساند. یعنی،

$$\left[\frac{\partial}{\partial E_1}\Omega\right]_{E_1=\bar{E}_1} = 0$$

$$\frac{\partial \Omega_1(E_1)}{\partial E_1}\Omega_2(E_2) + \Omega_1(E_1)\frac{\partial \Omega_2(E_2)}{\partial E_1} = 0$$

آنسامبل میکروکانونیک

$$\frac{\partial \Omega_1(E_1)}{\partial E_1}\Omega_2(E_2) + \Omega_1(E_1)\frac{\partial \Omega_2(E_2)}{\partial E_2}\frac{\partial E_2}{\partial E_1} = 0$$

$$E_2 = E - E_1$$

$$\frac{\partial \Omega_1(E_1)}{\partial E_1}\Omega_2(E_2) - \Omega_1(E_1)\frac{\partial \Omega_2(E_2)}{\partial E_2} = 0$$

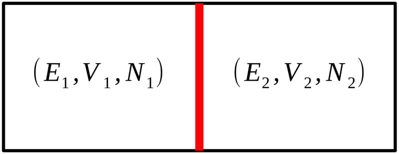
$$\div \Omega_1(E_1)\Omega_2(E_2) : \quad \frac{1}{\Omega_1(E_1)}\frac{\partial \Omega_1(E_1)}{\partial E_1} - \frac{1}{\Omega_2(E_2)}\frac{\partial \Omega_2(E_2)}{\partial E_2} = 0$$

$$\frac{1}{\Omega_1(E_1)}\frac{\partial \Omega_1(E_1)}{\partial E_1} = \frac{1}{\Omega_2(E_2)}\frac{\partial \Omega_2(E_2)}{\partial E_2}$$

$$\frac{1}{\Omega(E)}\frac{\partial \Omega(E)}{\partial E} = \text{const.} \Rightarrow \frac{1}{\Omega(E)}\frac{\partial \Omega(E)}{\partial E} = \frac{1}{k_B T} \Rightarrow \boxed{\frac{\partial}{\partial E}\ln \Omega(E) = \frac{1}{k_B T}}$$

آنسامبل میکروکانونیک

میکروحالت مرکب



$$\Omega(N_1, N_2) = \Omega_1(N_1)\Omega_2(N_2)$$

$$N = N_1 + N_2 \Rightarrow N_2 = N - N_1$$

$$\Omega(N, N_1) = \Omega_1(N_1)\Omega_2(N - N_1)$$

حالت تعادلی سیستم در N_1 وقتی اتفاق می‌افتد که بتواند مقدار میکروحالت مرکب را به حداکثر برساند. یعنی،

$$\left[\frac{\partial}{\partial N_1}\Omega\right]_{N_1=\bar{N}_1} = 0$$

$$\frac{\partial \Omega_1(N_1)}{\partial N_1}\Omega_2(N_2) + \Omega_1(N_1)\frac{\partial \Omega_2(N_2)}{\partial N_1} = 0$$

آنسامبل میکروکانونیک

$$\frac{\partial \Omega_1(N_1)}{\partial N_1}\Omega_2(N_2) + \Omega_1(N_1)\frac{\partial \Omega_2(N_2)}{\partial N_2}\frac{\partial N_2}{\partial N_1} = 0$$

$$N_2 = N - N_1$$

$$\frac{\partial \Omega_1(N_1)}{\partial N_1}\Omega_2(N_2) - \Omega_1(N_1)\frac{\partial \Omega_2(N_2)}{\partial N_2} = 0$$

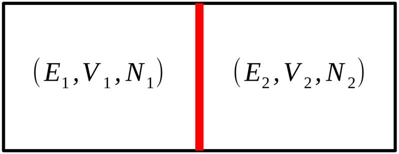
$$\div \Omega_1(N_1)\Omega_2(N_2): \quad \frac{1}{\Omega_1(N_1)}\frac{\partial \Omega_1(N_1)}{\partial N_1} - \frac{1}{\Omega_2(N_2)}\frac{\partial \Omega_2(N_2)}{\partial N_2} = 0$$

$$\frac{1}{\Omega_1(N_1)}\frac{\partial \Omega_1(N_1)}{\partial N_1} = \frac{1}{\Omega_2(N_2)}\frac{\partial \Omega_2(N_2)}{\partial N_2}$$

$$\frac{1}{\Omega(N)}\frac{\partial \Omega(N)}{\partial N} = \text{const.} \Rightarrow \frac{1}{\Omega(E)}\frac{\partial \Omega(N)}{\partial N} = -\frac{\mu}{k_B T} \Rightarrow \boxed{\frac{\partial}{\partial N}\ln \Omega(N) = -\frac{\mu}{k_B T}}$$

آنسامبل میکروکانونیک

میکروحالت مرکب



$$\Omega(V_1, V_2) = \Omega_1(V_1)\Omega_2(V_2)$$

$$V = V_1 + V_2 \Rightarrow V_2 = V - V_1$$

$$\Omega(V, V_1) = \Omega_1(V_1)\Omega_2(V - V_1)$$

حالت تعادلی سیستم در V_1 وقتی اتفاق می‌افتد که بتواند مقدار میکروحالت مرکب را به حداکثر برساند. یعنی،

$$\left[\frac{\partial}{\partial V_1}\Omega\right]_{V_1=\bar{V}_1} = 0$$

$$\frac{\partial \Omega_1(V_1)}{\partial V_1}\Omega_2(V_2) + \Omega_1(V_1)\frac{\partial \Omega_2(V_2)}{\partial V_1} = 0$$

آنسامبل میکروکانونیک

$$\frac{\partial \Omega_1(V_1)}{\partial V_1}\Omega_2(V_2) + \Omega_1(V_1)\frac{\partial \Omega_2(V_2)}{\partial V_2}\frac{\partial V_2}{\partial V_1} = 0$$

$$V_2 = V - V_1$$

$$\frac{\partial \Omega_1(V_1)}{\partial V_1}\Omega_2(V_2) - \Omega_1(V_1)\frac{\partial \Omega_2(V_2)}{\partial V_2} = 0$$

$$\div \Omega_1(V_1)\Omega_2(V_2): \quad \frac{1}{\Omega_1(V_1)}\frac{\partial \Omega_1(V_1)}{\partial V_1} - \frac{1}{\Omega_2(V_2)}\frac{\partial \Omega_2(V_2)}{\partial V_2} = 0$$

$$\frac{1}{\Omega_1(V_1)}\frac{\partial \Omega_1(V_1)}{\partial V_1} = \frac{1}{\Omega_2(V_2)}\frac{\partial \Omega_2(V_2)}{\partial V_2}$$

$$\frac{1}{\Omega(V)}\frac{\partial \Omega(V)}{\partial V} = \text{const.} \Rightarrow \frac{1}{\Omega(E)}\frac{\partial \Omega(V)}{\partial V} = -\frac{\mu}{k_B T} \Rightarrow \boxed{\frac{\partial}{\partial V}\ln \Omega(V) = \frac{p}{k_B T}}$$

آنسامبل میکروکانونیک

$$S = k_B \ln \Omega$$

$$\frac{\partial}{\partial E} \ln \Omega(E) = \frac{1}{k_B T} \qquad \left(\frac{\partial S}{\partial E}\right)_{N,V} = \frac{1}{T}$$

$$\frac{\partial}{\partial N} \ln \Omega(N) = -\frac{\mu}{k_B T} \qquad \left(\frac{\partial S}{\partial N}\right)_{V,E} = -\frac{\mu}{T}$$

$$\frac{\partial}{\partial V} \ln \Omega(V) = \frac{p}{k_B T} \qquad \left(\frac{\partial S}{\partial V}\right)_{N,E} = \frac{p}{T}$$

$$N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{E,N} + E\left(\frac{\partial S}{\partial E}\right)_{N,V} = S$$

$$-\frac{\mu N}{T} + \frac{pV}{T} + \frac{E}{T} = S \Rightarrow \boxed{-\mu N + pV + E = ST}$$

آنسامبل میکروکانونیک

$$n_1 + n2 = n, \quad n_1 = nf_1, \quad n_2 = nf_2$$
$$f_1 + f_2 = 1$$

$$C_p = C_V + Nk_B, \quad C_p/C_V = \gamma, \quad pV^\gamma = \text{const.}$$

$$Nk_B = nR$$

$$C_p = C_V + nR, \quad C_p/C_V = \gamma$$

$$C_V = \frac{nR}{\gamma - 1}$$
$$C_p = \frac{\gamma nR}{\gamma - 1}$$

آنسامبل میکروکانونیک

گاز نوع دوم

$$C_V^{(2)} = \frac{n_2 R}{\gamma_2 - 1}$$

$$C_p^{(2)} = \frac{\gamma_2 n_2 R}{\gamma_2 - 1}$$

$$n_2 = n f_2$$

گاز نوع اول

$$C_V^{(1)} = \frac{n_1 R}{\gamma_1 - 1},$$

$$C_p^{(1)} = \frac{\gamma_1 n_1 R}{\gamma_1 - 1},$$

$$n_1 = n f_1$$

گاز نوع اول + گاز نوع دوم

$$C_V = \frac{nR}{\gamma - 1}, \quad ①$$

$$C_p = \frac{\gamma n R}{\gamma - 1}, \quad ②$$

$$n_1 + n_2 = n$$

آنسامبل میکروکانونیک

گاز نوع اول

$$C_V^{(1)} = \frac{n_1 R}{\gamma_1 - 1},$$

$$C_p^{(1)} = \frac{\gamma_1 n_1 R}{\gamma_1 - 1},$$

$$n_1 = n f_1$$

گاز نوع دوم

$$C_V^{(2)} = \frac{n_2 R}{\gamma_2 - 1}$$

$$C_p^{(2)} = \frac{\gamma_2 n_2 R}{\gamma_2 - 1}$$

$$n_2 = n f_2$$

$$C_V = C_V^{(1)} + C_V^{(2)} = \frac{n_1 R}{\gamma_1 - 1} + \frac{n_2 R}{\gamma_2 - 1}$$

$$C_V = \frac{n f_1 R}{\gamma_1 - 1} + \frac{n f_2 R}{\gamma_2 - 1}$$

$$C_V = \left[\frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1}\right] nR$$

گاز نوع اول

$$C_V^{(1)} = \frac{n_1 R}{\gamma_1 - 1},$$

$$C_p^{(1)} = \frac{\gamma_1 n_1 R}{\gamma_1 - 1},$$

$$n_1 = n f_1$$

گاز نوع دوم

$$C_V^{(2)} = \frac{n_2 R}{\gamma_2 - 1}$$

$$C_p^{(2)} = \frac{\gamma_2 n_2 R}{\gamma_2 - 1}$$

$$n_2 = n f_2$$

$$C_V = C_V^{(1)} + C_V^{(2)} = \left[\frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1}\right] nR, \quad ③$$

$$① \text{ and } ③ \Rightarrow \boxed{\frac{1}{\gamma - 1} = \frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1}}$$

گاز نوع اول

$$C_V^{(1)} = \frac{n_1 R}{\gamma_1 - 1},$$

$$C_p^{(1)} = \frac{\gamma_1 n_1 R}{\gamma_1 - 1},$$

$$n_1 = n f_1$$

گاز نوع دوم

$$C_V^{(2)} = \frac{n_2 R}{\gamma_2 - 1}$$

$$C_p^{(2)} = \frac{\gamma_2 n_2 R}{\gamma_2 - 1}$$

$$n_2 = n f_2$$

$$C_p = C_p^{(1)} + C_p^{(2)} = \left[\frac{\gamma_1 f_1}{\gamma_1 - 1} + \frac{\gamma_2 f_2}{\gamma_2 - 1}\right] nR, \quad \text{(۴)}$$

$$\text{(۲) and (۴)} \Rightarrow \frac{\gamma}{\gamma - 1} = \frac{\gamma_1 f_1}{\gamma_1 - 1} + \frac{\gamma_2 f_2}{\gamma_2 - 1} \Rightarrow \frac{\gamma}{\gamma - 1} = \frac{\gamma_1 f_1}{\gamma_1 - 1} + \frac{\gamma_2 f_2}{\gamma_2 - 1}$$

$$\frac{-1+1+\gamma}{\gamma - 1} = \frac{(-1+1+\gamma_1) f_1}{\gamma_1 - 1} + \frac{(-1+1+\gamma_2) f_2}{\gamma_2 - 1}$$

گاز نوع اول

$$C_V^{(1)} = \frac{n_1 R}{\gamma_1 - 1},$$

$$C_p^{(1)} = \frac{\gamma_1 n_1 R}{\gamma_1 - 1},$$

$$n_1 = n f_1$$

گاز نوع دوم

$$C_V^{(2)} = \frac{n_2 R}{\gamma_2 - 1}$$

$$C_p^{(2)} = \frac{\gamma_2 n_2 R}{\gamma_2 - 1}$$

$$n_2 = n f_2$$

$$\frac{-1+1+\gamma}{\gamma - 1} = \frac{(-1+1+\gamma_1)f_1}{\gamma_1 - 1} + \frac{(-1+1+\gamma_2)f_2}{\gamma_2 - 1}$$

$$\frac{1}{\gamma - 1} + 1 = \frac{f_1}{\gamma_1 - 1} + f_1 + \frac{f_2}{\gamma_2 - 1} + f_2$$

$$\frac{1}{\gamma - 1} + 1 = \frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1} + f_1 + f_2 \Rightarrow \boxed{\frac{1}{\gamma - 1} = \frac{f_1}{\gamma_1 - 1} + \frac{f_2}{\gamma_2 - 1}}$$

آنسامبل میکروکانونیک

درستی روابط زیر را بررسی کنید

$$\frac{S}{V} = \left(\frac{\partial p}{\partial T}\right)_\mu, \quad \frac{N}{V} = \left(\frac{\partial p}{\partial \mu}\right)_T$$

از قانون اول ترمودینامیک داریم،

$$\mathrm{d}E = -p\mathrm{d}V + T\mathrm{d}S + \mu\mathrm{d}N, \quad (*)$$

از طرفی در مسئله اول بدست آوردیم،

$$E = -pV + \mu N + ST$$

اگر از رابطه بالا دیفرانسیل گرفته شود،

$$\mathrm{d}E = -p\mathrm{d}V - V\mathrm{d}p + T\mathrm{d}S + S\mathrm{d}T + \mu\mathrm{d}N + N\mathrm{d}\mu$$

با استفاده از رابطه $(*)$ می‌توان جملات قرمز رنگ سمت راست و چپ را کنسل کرد،

$$\color{red}{\mathrm{d}E} = \color{red}{-p\mathrm{d}V} - V\mathrm{d}p + \color{red}{T\mathrm{d}S} + S\mathrm{d}T + \color{red}{\mu\mathrm{d}N} + N\mathrm{d}\mu$$

آنسامبل میکروکانونیک

$$\mathrm{d}E = -p\mathrm{d}V - V\mathrm{d}p + T\mathrm{d}S + S\mathrm{d}T + \mu\mathrm{d}N + N\mathrm{d}\mu$$

بنابراین

$$0 = -V\mathrm{d}p + S\mathrm{d}T + N\mathrm{d}\mu$$

$$V\mathrm{d}p = S\mathrm{d}T + N\mathrm{d}\mu$$

$$\mathrm{d}p = \frac{S}{V}\mathrm{d}T + \frac{N}{V}\mathrm{d}\mu, \quad \text{(I)}$$

اگر $p = p(T, \mu)$ فرض کنیم

$$\mathrm{d}p = \left(\frac{\partial p}{\partial T}\right)_\mu \mathrm{d}T + \left(\frac{\partial p}{\partial \mu}\right)_T \mathrm{d}\mu, \quad \text{(II)}$$

آنسامبل میکروکانونیک

$$\mathrm{d}p = \frac{S}{V}\mathrm{d}T + \frac{N}{V}\mathrm{d}\mu, \quad \textcircled{I}$$

$$\mathrm{d}p = \left(\frac{\partial p}{\partial T}\right)_{\mu}\mathrm{d}T + \left(\frac{\partial p}{\partial \mu}\right)_{T}\mathrm{d}\mu, \quad \textcircled{II}$$

اگر سمت چپ دو عبارت (I) و (II) را باهم مقایسه کنیم

$$\frac{S}{V} = \left(\frac{\partial p}{\partial T}\right)_{\mu}, \quad \frac{N}{V} = \left(\frac{\partial p}{\partial \mu}\right)_{T}$$

آنسامبل میکروکانونیک

$$S = Nk_B\left(\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]+\frac{5}{2}\right)$$

$$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_{N,E} = Nk_B\frac{1}{V} \Rightarrow pV = Nk_BT$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = -\frac{\mu}{T}$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = k_B\left(\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]+\frac{5}{2}\right) + Nk_B\frac{-\frac{V}{N^2}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}-\frac{3}{2}\frac{V}{N}N^{-5/2}\left(\frac{4m\pi E}{3h^2}\right)^{3/2}}{\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}}$$

آنسامبل میکروکانونیک

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = -\frac{\mu}{T}$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = k_B\left(\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right) + Nk_B\frac{-\frac{V}{N^2}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2} - \frac{3}{2}\frac{V}{N^2}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}}{\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}}$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = k_B\left(\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right) - \frac{5}{2}k_B$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = k_B\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]$$

آنسامبل میکروکانونیک

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = -\frac{\mu}{T}$$

$$\left(\frac{\partial S}{\partial N}\right)_{V,E} = k_B \ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]$$

$$-\frac{\mu}{T} = k_B \ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right], \quad \frac{V}{N} = \frac{k_B T}{p}, \quad \frac{2E}{3N} = k_B T \qquad (۵)$$

$$e^{-\mu/k_B T} = \frac{k_B T}{p}\left(\frac{2m\pi k_B T}{h^2}\right)^{3/2}$$

$$p = \left(\frac{2m\pi}{h^2}\right)^{3/2}(k_B T)^{5/2} e^{\mu/k_B T}$$

آنسامبل میکروکانونیک

$$p = \left(\frac{2m\pi}{h^2}\right)^{3/2} (k_B T)^{5/2} e^{\mu/k_B T}$$

$$\frac{S}{V} = \left(\frac{\partial p}{\partial T}\right)_\mu, \quad \frac{N}{V} = \left(\frac{\partial p}{\partial \mu}\right)_T$$

$$\left(\frac{\partial p}{\partial \mu}\right)_T = \left(\frac{2m\pi}{h^2}\right)^{3/2} (k_B T)^{5/2} \frac{1}{k_B T} e^{\mu/k_B T} = \frac{p}{k_B T}, \quad pV = N k_B T$$

$$\left(\frac{\partial p}{\partial \mu}\right)_T = \frac{p}{k_B T} = \frac{N}{V}$$

$$\left(\frac{\partial p}{\partial T}\right)_\mu = \left(\frac{2m\pi}{h^2}\right)^{3/2} \left((k_B)^{5/2} \frac{5}{2} T^{3/2} - (k_B T)^{5/2} \frac{\mu}{T^2}\right) e^{\mu/k_B T}$$

آنسامبل میکروکانونیک

$$p = \left(\frac{2m\pi}{h^2}\right)^{3/2} (k_B T)^{5/2} e^{\mu/k_B T}$$

$$\frac{S}{V} = \left(\frac{\partial p}{\partial T}\right)_\mu, \quad \frac{N}{V} = \left(\frac{\partial p}{\partial \mu}\right)_T$$

$$\left(\frac{\partial p}{\partial T}\right)_\mu = \left(\frac{2m\pi}{h^2}\right)^{3/2} \left(\frac{5}{2}(k_B)^{5/2} T^{3/2} - (k_B T)^{5/2} \frac{\mu}{k_B T^2}\right) e^{\mu/k_B T}$$

$$\left(\frac{\partial p}{\partial T}\right)_\mu = \frac{5}{2}\frac{p}{T} - \frac{p\mu}{k_B T^2} = \frac{p}{T}\left(\frac{5}{2} - \frac{\mu}{k_B T}\right)$$

با استفاده از رابطه (۵)

$$-\frac{\mu}{k_B T} = \ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]$$

آنسامبل میکروکانونیک

$$p = \left(\frac{2m\pi}{h^2}\right)^{3/2} (k_B T)^{5/2} e^{\mu/k_B T}$$

$$\frac{S}{V} = \left(\frac{\partial p}{\partial T}\right)_\mu, \quad \frac{N}{V} = \left(\frac{\partial p}{\partial \mu}\right)_T$$

$$\left(\frac{\partial p}{\partial T}\right)_\mu = \frac{p}{T}\left(\frac{5}{2} + \ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right]\right)$$

با استفاده از رابطه انتروپی بدست آمده از آنسامبل میکروکانونیک

$$\frac{S}{Nk_B} = \left(\ln\left[\frac{V}{N}\left(\frac{4m\pi E}{3Nh^2}\right)^{3/2}\right] + \frac{5}{2}\right)$$

$$\left(\frac{\partial p}{\partial T}\right)_\mu = \frac{p}{T}\frac{S}{Nk_B}, \text{ and } pV = Nk_B T \Rightarrow \left(\frac{\partial p}{\partial T}\right)_\mu = \frac{S}{V}$$