

# فیزیک ۱

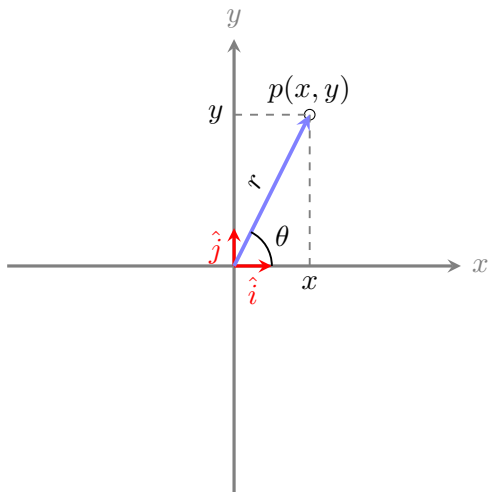
## دستگاه مختصات قطبی

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گروه فیزیک، دانشکده علوم پایه، دانشگاه قم

بهمن ۱۴۰۰

# بردارهای یکه



بردار یکه

$$|\hat{i}| = |\hat{j}| = 1, \quad \hat{i} \cdot \hat{j} = 0$$

بردار مکان

$$\vec{r} = x\hat{i} + y\hat{j}$$

اندازه

$$r = |\vec{r}|$$

$$r = \sqrt{\vec{r} \cdot \vec{r}} = \sqrt{x^2 + y^2}$$

# بردارهای یکه

بردار یکه

$$|\hat{i}| = |\hat{j}| = 1, \quad \hat{i} \cdot \hat{j} = 0$$

بردار مکان

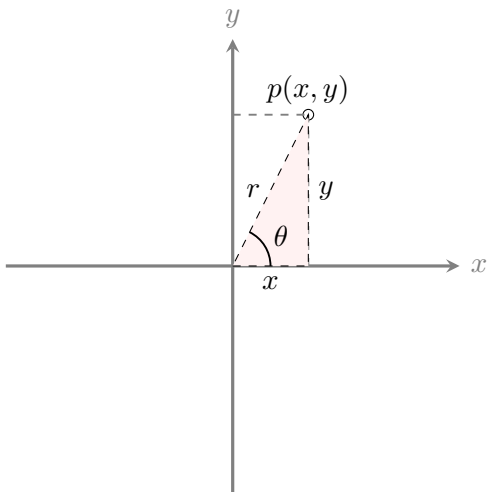
$$\vec{r} = x\hat{i} + y\hat{j}$$

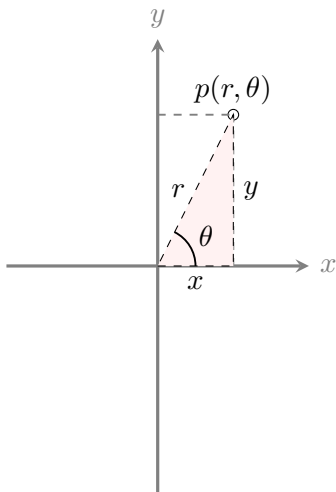
اندازه

$$r = \sqrt{x^2 + y^2}$$

جهت

$$\tan \theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1} \left( \frac{y}{x} \right)$$





اندازه و جهت

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1} \left( \frac{y}{x} \right)$$

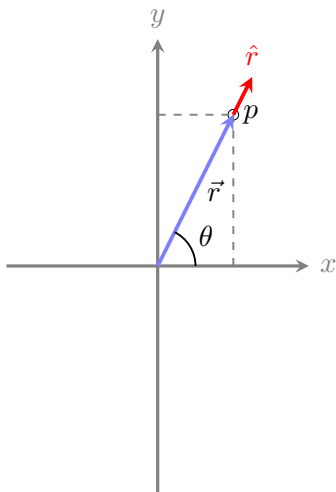
بردار مکان

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$\vec{r} = \hat{i}x + \hat{j}y$$

$$\vec{r} = \hat{i}r \cos \theta + \hat{j}r \sin \theta$$

$$\vec{r} = r(\hat{i} \cos \theta + \hat{j} \sin \theta)$$



اندازه و جهت

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1} \left( \frac{y}{x} \right)$$

بردار مکان

$$\vec{r} = r(\hat{i} \cos \theta + \hat{j} \sin \theta)$$

$$\vec{r} = r\hat{r} \Rightarrow \hat{r} = \frac{\vec{r}}{r}$$

$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$$

$$|\hat{r}| = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$$

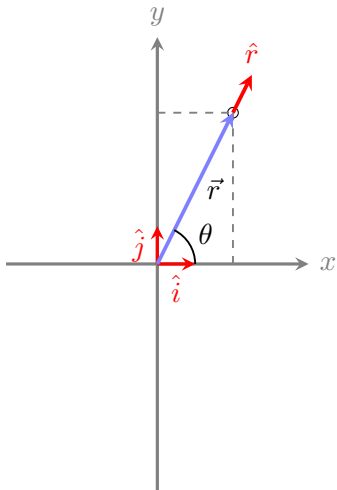
# بردارهای یکه

## بردار مکان

$$\vec{r} = r\hat{r}, \quad |\hat{r}| = 1$$

## بردار یکه

$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$$



◀ بطور کلی برای هر دستگاه مختصات دوبعدی دو بردار یکه وجود دارد. برای مثال،  $\hat{i}$  و  $\hat{j}$ ، در دستگاه مختصات دکارتی در دو بعد بردارهای یکه می‌باشند.

◀ به همین ترتیب برای دستگاه مختصات قطبی، علاوه بر بردار یکه  $\hat{r}$ ، نیاز به یک بردار یکه دیگر نیز می‌باشد.

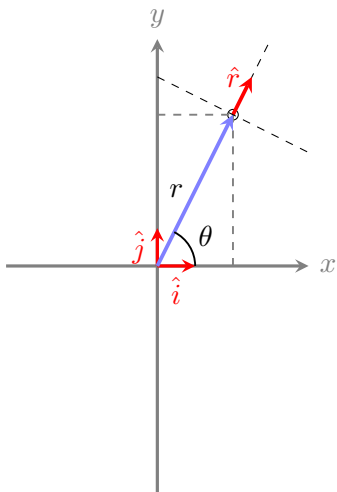
# بردارهای یکه

بردار مکان

$$\vec{r} = r\hat{r}, \quad |\hat{r}| = 1$$

بردار یکه

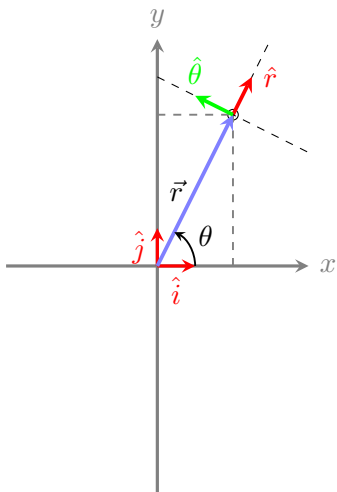
$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$$



بردارهای یکه بر یکدیگر عمود هستند. برای مثال در دستگاه مختصات دکارتی، بردارهای یکه  $\hat{i}$  و  $\hat{j}$  بر یکدیگر عمودند.

$$\hat{i} \cdot \hat{j} = 0$$

به همین ترتیب برای دستگاه مختصات قطبی، بردار یکه‌ای وجود دارد که بر بردار یکه  $\hat{r}$  عمود است.



بردار مکان

$$\vec{r} = r\hat{r}, \quad |\hat{r}| = 1$$

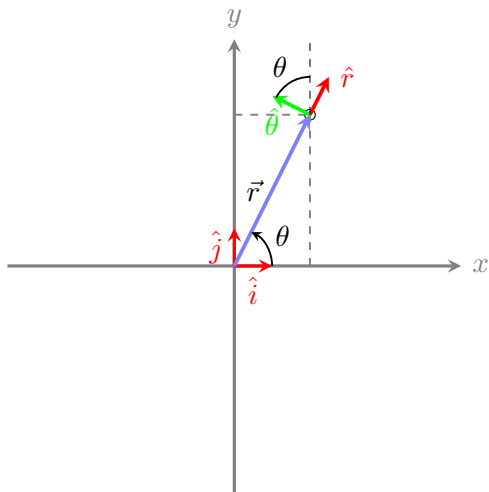
بردار یکه

$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$$

◀ بردارهای یکه در امتداد مثبت تغییرات مختصات هستند. برای مثال در دستگاه مختصات دکارتی، بردار یکه  $\hat{i}$  در امتداد مثبت تغییرات محور  $x$  است و بردار یکه  $\hat{j}$  در امتداد مثبت تغییرات محور  $y$  است.

◀ در دستگاه مختصات قطبی،  $\hat{r}$  در امتداد مثبت تغییرات بردار  $\vec{r}$  است.





بردار مکان

$$\vec{r} = r\hat{r}, \quad |\hat{r}| = 1$$

بردار یکه

$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta$$

$$\hat{\theta} = -\hat{i} \sin \theta + \hat{j} \cos \theta$$

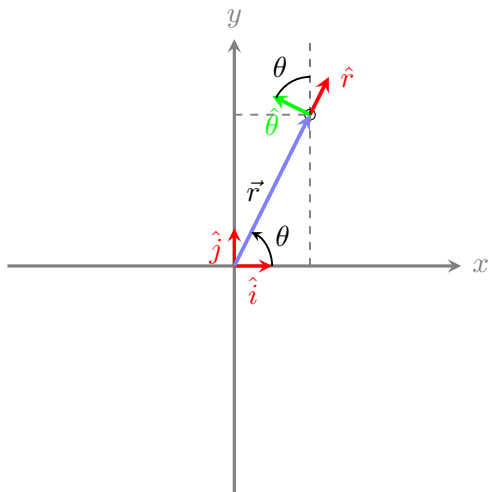
$$|\hat{\theta}| = 1$$

$$\sqrt{(-\sin \theta)^2 + \cos^2 \theta} = 1$$

$$\hat{r} \cdot \hat{\theta} = 0$$

$$\cos \theta (-\sin \theta) + \cos \theta \sin \theta = 0$$

# بردارهای یکه



بردار مکان

$$\vec{r} = r\hat{r}, \quad |\hat{r}| = 1$$

بردار یکه

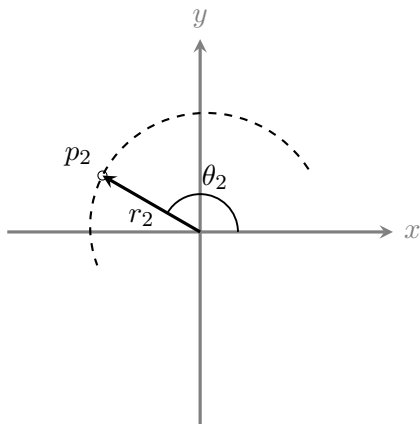
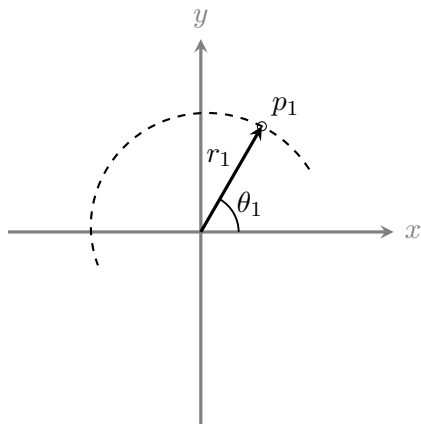
$$\hat{r} = \hat{i} \cos \theta + \hat{j} \sin \theta, \quad |\hat{r}| = 1$$

$$\hat{\theta} = -\hat{i} \sin \theta + \hat{j} \cos \theta, \quad |\hat{\theta}| = 1$$

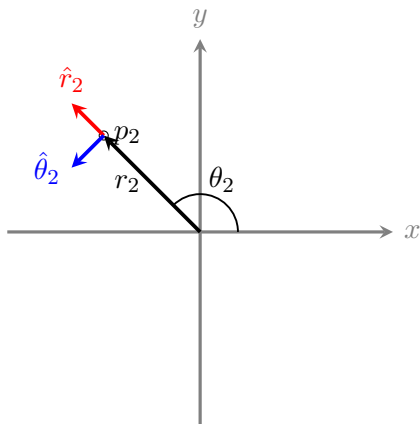
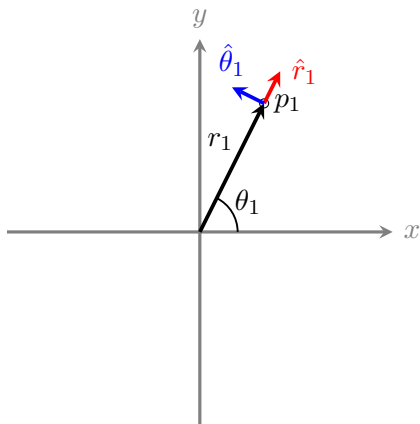
$$\hat{r} \cdot \hat{\theta} = 0$$

$$\hat{r} \times \hat{\theta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \end{vmatrix}$$

$$\hat{r} \times \hat{\theta} = \hat{k}$$



$$r = r(t), \theta = \theta(t) : \begin{cases} p_1 : r_1 = r(t_1), \theta_1 = \theta(t_1) \\ p_2 : r_2 = r(t_2), \theta_2 = \theta(t_2) \end{cases}$$



$$r = r(t), \theta = \theta(t) : \begin{cases} p_1 : r_1 = r(t_1), \theta_1 = \theta(t_1), \hat{r}_1 = \hat{r}(\theta_1), \hat{\theta}_1 = \hat{\theta}(\theta_1) \\ p_2 : r_2 = r(t_2), \theta_2 = \theta(t_2), \hat{r}_2 = \hat{r}(\theta_2), \hat{\theta}_2 = \hat{\theta}(\theta_2) \end{cases}$$

$$r = r(t), \quad \theta = \theta(t), \quad \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\vec{r} = r \hat{r}$$

$$\frac{d\hat{r}}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} = \hat{\theta}$$

$$\frac{d}{dt} \vec{r} = \frac{d}{dt} (r \hat{r})$$

$$\vec{v} = \frac{dr}{dt} \hat{r} + r \hat{\theta} \frac{d\theta}{dt}$$

$$\vec{v} = \frac{dr}{dt} \hat{r} + r \frac{d\hat{r}}{dt}$$

$$\dot{r} = \frac{dr}{dt}, \quad \dot{\theta} = \frac{d\theta}{dt}$$

اگر

قاعده‌ی زنجیری

$$\vec{v} = \frac{dr}{dt} \hat{r} + r \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt}$$

$$\boxed{\vec{v} = \dot{r} \hat{r} + r \dot{\theta} \hat{\theta}}$$

$$r = r(t), \quad \theta = \theta(t), \quad \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

قاعده زنجیری

$$\vec{v} = v_r \hat{r} + v_\theta \hat{\theta}$$

$$\begin{cases} v_r = \dot{r} \\ v_\theta = r\dot{\theta} \end{cases}$$

$$\frac{d}{dt} \vec{v} = \frac{d}{dt} (\dot{r} \hat{r}) + \frac{d}{dt} (r \dot{\theta} \hat{\theta})$$

$$\begin{aligned} \vec{a} &= \frac{d\dot{r}}{dt} \hat{r} + \dot{r} \frac{d\hat{r}}{dt} \\ &+ \frac{dr}{dt} \dot{\theta} \hat{\theta} + r \frac{d\dot{\theta}}{dt} \hat{\theta} + r \dot{\theta} \frac{d\hat{\theta}}{dt} \end{aligned}$$

$$\begin{aligned} \vec{a} &= \frac{d\dot{r}}{dt} \hat{r} + \dot{r} \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt} \\ &+ \frac{dr}{dt} \dot{\theta} \hat{\theta} + r \frac{d\dot{\theta}}{dt} \hat{\theta} + r \dot{\theta} \frac{d\hat{\theta}}{d\theta} \frac{d\theta}{dt} \end{aligned}$$

اگر

$$\dot{r} = \frac{dr}{dt}, \quad \dot{\theta} = \frac{d\theta}{dt}$$

$$\ddot{r} = \frac{d\dot{r}}{dt}, \quad \ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

$$r = r(t), \quad \theta = \theta(t), \quad \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\begin{aligned} \vec{a} &= \frac{d\dot{r}}{dt} \hat{r} + \dot{r} \frac{d\hat{r}}{d\theta} \frac{d\theta}{dt} \\ &+ \frac{dr}{dt} \dot{\theta} \hat{\theta} + r \frac{d\dot{\theta}}{dt} \hat{\theta} + r \dot{\theta} \frac{d\hat{\theta}}{d\theta} \frac{d\theta}{dt} \end{aligned}$$

$$\begin{aligned} \vec{a} &= \ddot{r} \hat{r} + \dot{r} \dot{\theta} \frac{d\hat{r}}{d\theta} \\ &+ \dot{r} \dot{\theta} \hat{\theta} + r \ddot{\theta} \hat{\theta} + r \dot{\theta}^2 \frac{d\hat{\theta}}{d\theta} \end{aligned}$$

اگر

$$\dot{r} = \frac{dr}{dt}, \quad \dot{\theta} = \frac{d\theta}{dt}$$

$$\ddot{r} = \frac{d\dot{r}}{dt}, \quad \ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

$$\frac{d\hat{r}}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} = \hat{\theta}$$

$$\frac{d\hat{\theta}}{d\theta} = -\cos \theta \hat{i} - \sin \theta \hat{j} = -\hat{r}$$

$$r = r(t), \quad \theta = \theta(t), \quad \hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\begin{aligned} \vec{a} &= \ddot{r} \hat{r} + \dot{r} \dot{\hat{r}} \\ &\quad + \dot{r} \dot{\hat{\theta}} + r \ddot{\hat{\theta}} + r \dot{\theta}^2 \frac{d\hat{\theta}}{d\theta} \end{aligned}$$

$$\frac{d\hat{r}}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} = \hat{\theta}$$

$$\frac{d\hat{\theta}}{d\theta} = -\cos \theta \hat{i} - \sin \theta \hat{j} = -\hat{r}$$

$$\begin{aligned} \vec{a} &= \ddot{r} \hat{r} + \dot{r} \dot{\hat{\theta}} \\ &\quad + \dot{r} \dot{\hat{\theta}} + r \ddot{\hat{\theta}} - r \dot{\theta}^2 \hat{r} \end{aligned}$$

$$\vec{a} = (\ddot{r} - r \dot{\theta}^2) \hat{r} + (2\dot{r} \dot{\theta} + r \ddot{\theta}) \hat{\theta}$$

$$\vec{a} = a_r \hat{r} + a_\theta \hat{\theta}$$

$$\begin{cases} a_r = \ddot{r} - r \dot{\theta}^2 \\ a_\theta = 2\dot{r} \dot{\theta} + r \ddot{\theta} \end{cases}$$



$$r = r(t), \quad \theta = \theta(t)$$

$$\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\frac{d\hat{r}}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} = \hat{\theta}, \quad \frac{d\hat{\theta}}{d\theta} = -\cos \theta \hat{i} - \sin \theta \hat{j} = -\hat{r}$$

بطور خلاصه

حالت‌های خاص ۱- حرکت بر مسیر دایره‌ای با شعاع ثابت  $R$

$$r = r(t), \quad \theta = \theta(t)$$

$$\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\vec{r} = r\hat{r}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$r = R, \quad \theta = \theta(t)$$

$$\vec{r} = R\hat{r}$$

$$\vec{v} = R\dot{\theta}\hat{\theta}$$

$$\vec{a} = -R\dot{\theta}^2\hat{r} + R\ddot{\theta}\hat{\theta}$$

$$r = r(t), \quad \theta = \theta(t)$$

$$\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}, \quad \hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\frac{d\hat{r}}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} = \hat{\theta}, \quad \frac{d\hat{\theta}}{d\theta} = -\cos \theta \hat{i} - \sin \theta \hat{j} = -\hat{r}$$

بطور خلاصه

حالت‌های خاص ۲- حرکت بر مسیر دایره‌ای با شعاع ثابت  $R$  و سرعت زاویه‌ای ثابت  $\omega$

$$r = r(t), \quad \theta = \theta(t)$$

$$\hat{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\vec{r} = r\hat{r}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$r = R, \quad \theta = \omega t + \theta_0$$

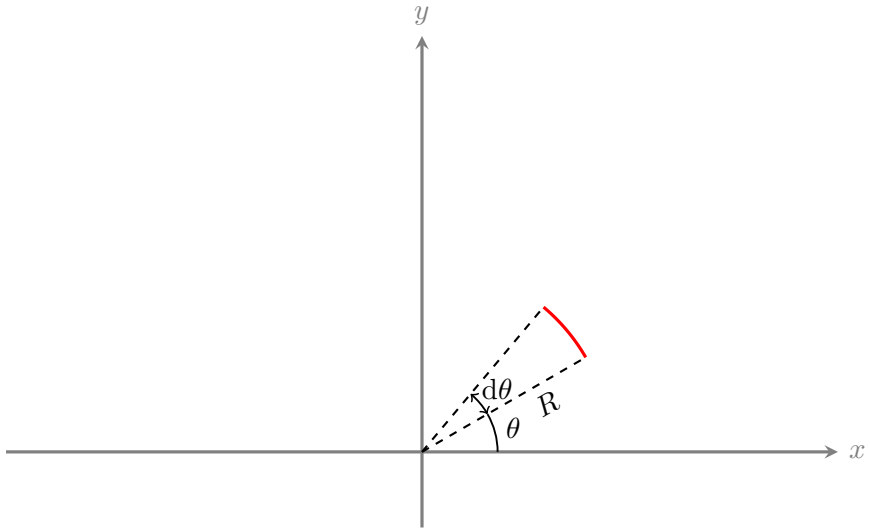
$$\vec{r} = R\hat{r}$$

$$\vec{v} = R\omega\hat{\theta}$$

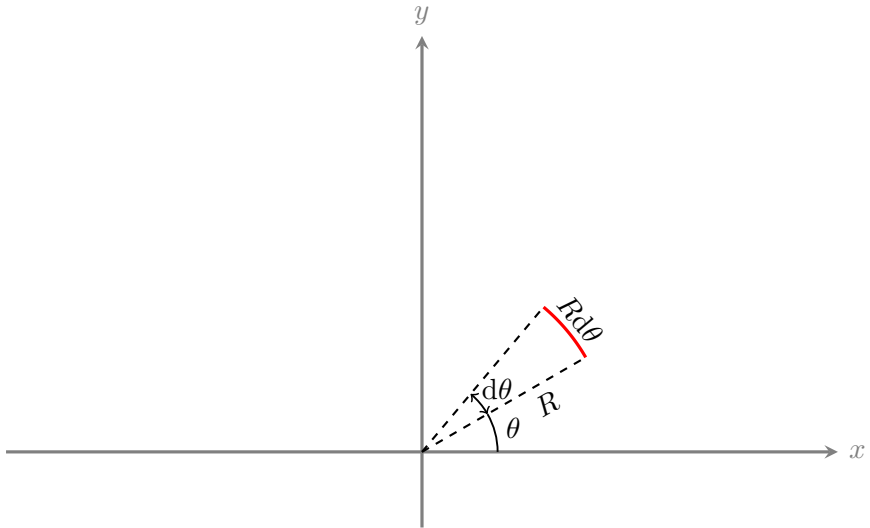
$$\vec{a} = -R\dot{\theta}^2\hat{r}$$

سرعت مماسی و شتاب شعاعی

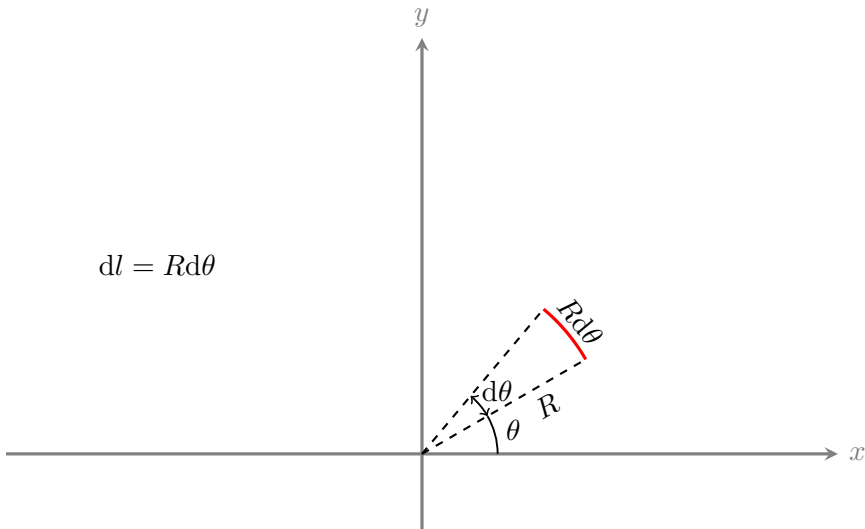
# انتگرالگیری یک بعدی



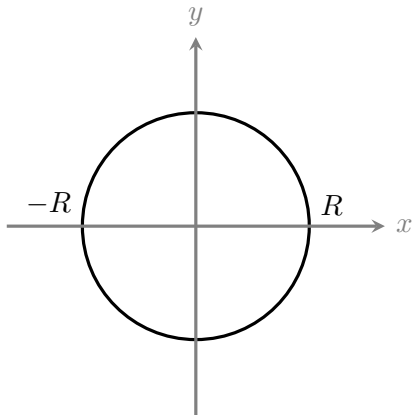
# انتگرالگیری یک بعدی



# انتگرالگیری یک بعدی



## انتگرالگیری یک بعدی



$$dl = R d\theta$$

$$0 \leq \theta \leq 2\pi$$

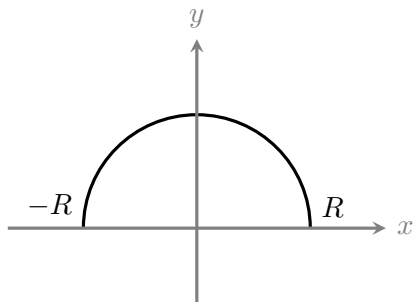
$$l = \int_0^{2\pi} R d\theta$$

$$l = R \left( \int_0^{2\pi} d\theta \right)$$

$$l = R \left( [\theta]_0^{2\pi} \right)$$

$$l = R \times 2\pi = 2\pi R$$

## انتگرالگیری یک بعدی



$$dl = R d\theta$$

$$0 \leq \theta \leq \pi$$

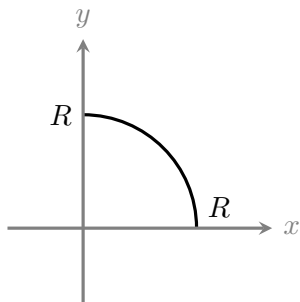
$$l = \int_0^{\pi} R d\theta$$

$$l = R \left( \int_0^{\pi} d\theta \right)$$

$$l = R ([\theta]_0^{\pi})$$

$$l = R \times \pi = \pi R$$

## انتگرالگیری یک بعدی



$$dl = R d\theta$$

$$0 \leq \theta \leq \pi/2$$

$$l = \int_0^{\pi/2} R d\theta$$

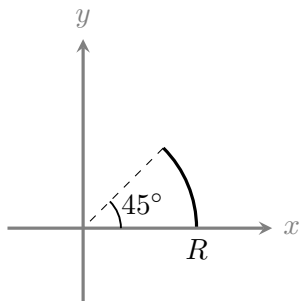
$$l = R \left( \int_0^{\pi/2} d\theta \right)$$

$$l = R \left( [\theta]_0^{\pi/2} \right)$$

$$l = R \times \frac{\pi}{2} = \frac{1}{2} \pi R$$



## انتگرالگیری یک بعدی



$$dl = R d\theta$$

$$0 \leq \theta \leq \pi/4$$

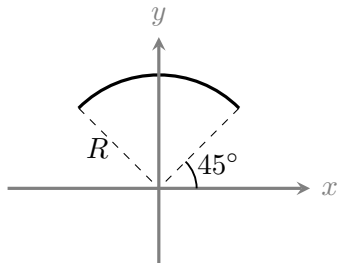
$$l = \int_0^{\pi/4} R d\theta$$

$$l = R \left( \int_0^{\pi/4} d\theta \right)$$

$$l = R \left( [\theta]_0^{\pi/4} \right)$$

$$l = R \times \frac{\pi}{4} = \frac{1}{4} \pi R$$

# انتگرالگیری یک بعدی



$$dl = R d\theta$$

$$\pi/4 \leq \theta \leq 3\pi/4$$

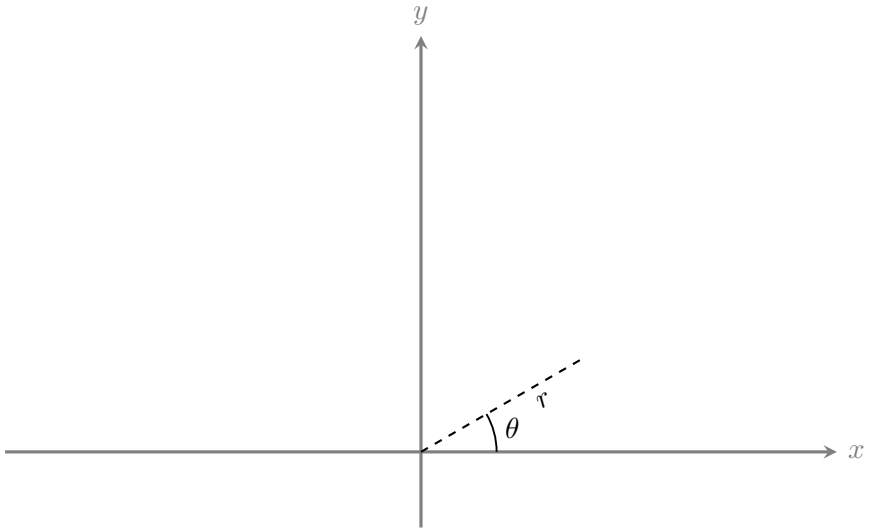
$$l = \int_{\pi/4}^{3\pi/4} R d\theta$$

$$l = R \left( \int_{\pi/4}^{3\pi/4} d\theta \right)$$

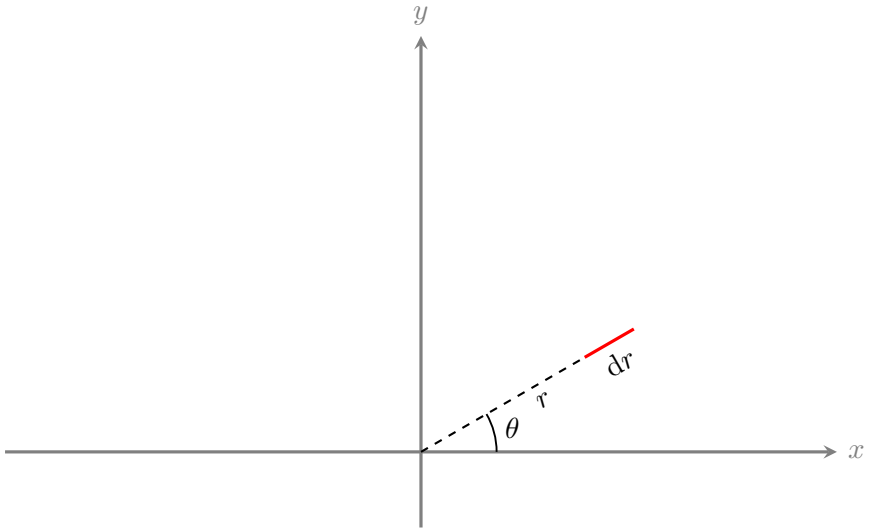
$$l = R \left( [\theta]_{\pi/4}^{3\pi/4} \right)$$

$$l = R \times \frac{\pi}{4} = \frac{1}{4} \pi R$$

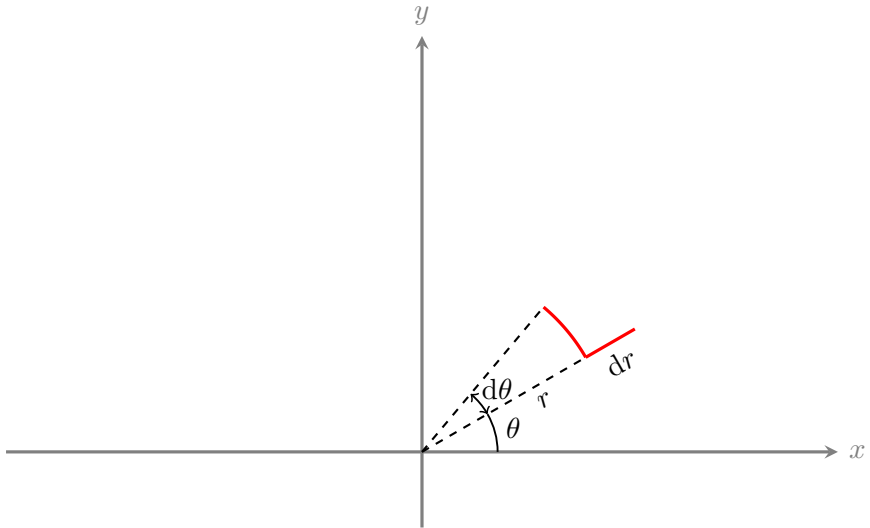
# انتگرالگیری دوبعدی



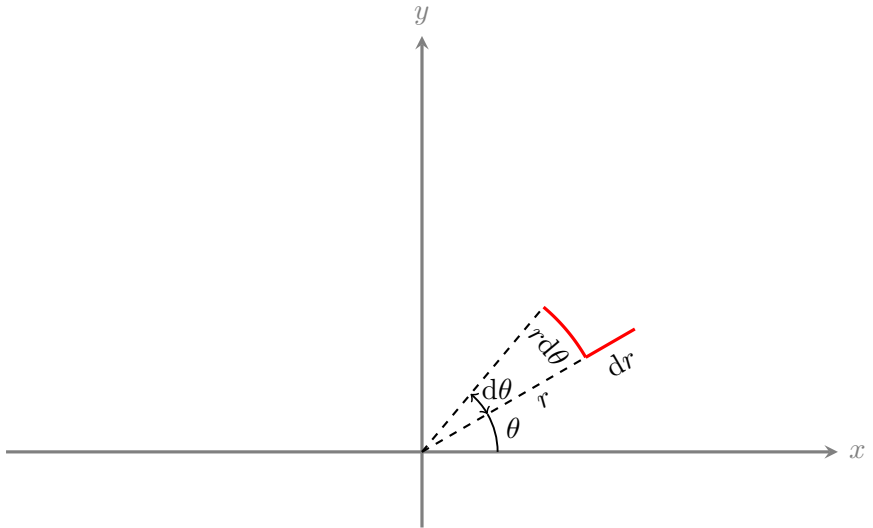
# انتگرالگیری دوبعدی



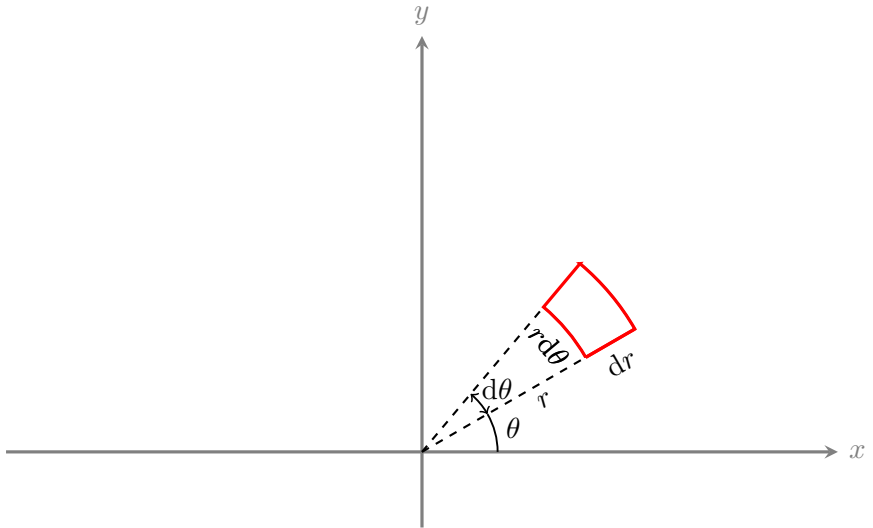
# انتگرالگیری دوبعدی



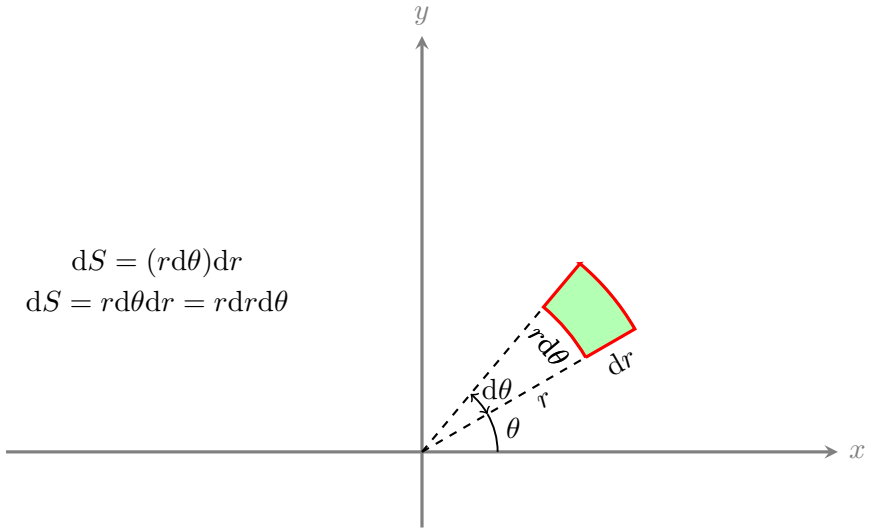
# انتگرالی دو بعدی



# انتگرالی دو بعدی

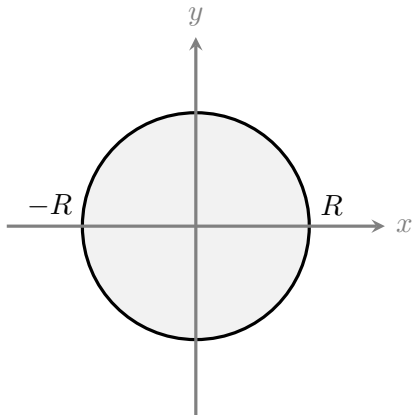


# انتگرالی دو بعدی



$$dS = (rd\theta)dr$$
$$dS = rd\theta dr = r dr d\theta$$





$$dS = r dr d\theta$$

$$0 \leq r \leq R$$

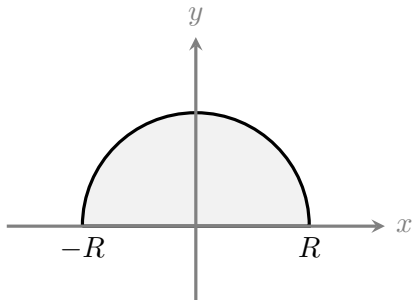
$$0 \leq \theta \leq 2\pi$$

$$S = \int_0^R \int_0^{2\pi} r dr d\theta$$

$$S = \left( \int_0^R r dr \right) \left( \int_0^{2\pi} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_0^R \right) \left( [\theta]_0^{2\pi} \right)$$

$$S = \frac{1}{2} R^2 \times 2\pi = \pi R^2$$



$$dS = r dr d\theta$$

$$0 \leq r \leq R$$

$$0 \leq \theta \leq \pi$$

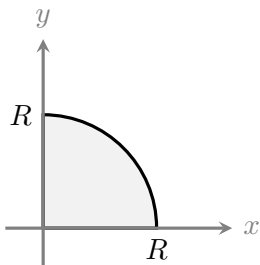
$$S = \int_0^R \int_0^\pi r dr d\theta$$

$$S = \left( \int_0^R r dr \right) \left( \int_0^\pi d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_0^R \right) ([\theta]_0^\pi)$$

$$S = \frac{1}{2} R^2 \times \pi = \frac{1}{2} \pi R^2$$

# انتگرالگیری دوبعدی



$$dS = r dr d\theta$$

$$0 \leq r \leq R$$

$$0 \leq \theta \leq \pi/2$$

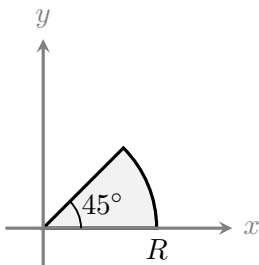
$$S = \int_0^R \int_0^{\pi/2} r dr d\theta$$

$$S = \left( \int_0^R r dr \right) \left( \int_0^{\pi/2} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_0^R \right) \left( [\theta]_0^{\pi/2} \right)$$

$$S = \frac{1}{2} R^2 \times \frac{\pi}{2} = \frac{1}{4} \pi R^2$$

# انتگرالی دو بعدی



$$dS = r dr d\theta$$

$$0 \leq r \leq R$$

$$0 \leq \theta \leq \pi/4$$

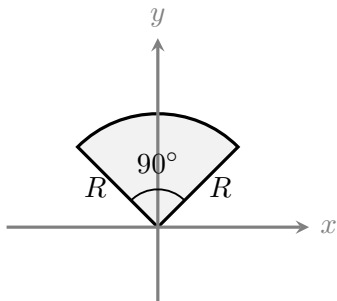
$$S = \int_0^R \int_0^{\pi/4} r dr d\theta$$

$$S = \left( \int_0^R r dr \right) \left( \int_0^{\pi/4} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_0^R \right) \left( [\theta]_0^{\pi/4} \right)$$

$$S = \frac{1}{2} R^2 \times \frac{\pi}{4} = \frac{1}{8} \pi R^2$$

# انتگرالی دوعدی



$$dS = r dr d\theta$$

$$0 \leq r \leq R$$

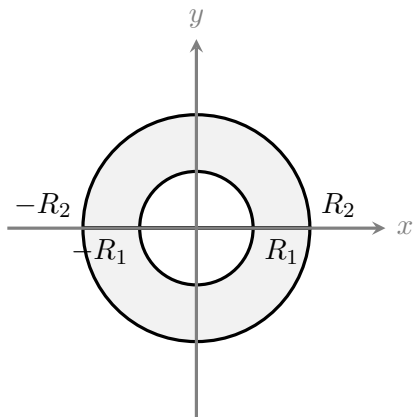
$$\pi/4 \leq \theta \leq 3\pi/4$$

$$S = \int_0^R \int_{\pi/4}^{3\pi/4} r dr d\theta$$

$$S = \left( \int_0^R r dr \right) \left( \int_{\pi/4}^{3\pi/4} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_0^R \right) \left( [\theta]_{\pi/4}^{3\pi/4} \right)$$

$$S = \frac{1}{2} R^2 \times \frac{\pi}{2} = \frac{1}{4} \pi R^2$$



$$dS = r dr d\theta$$

$$R_1 \leq r \leq R_2$$

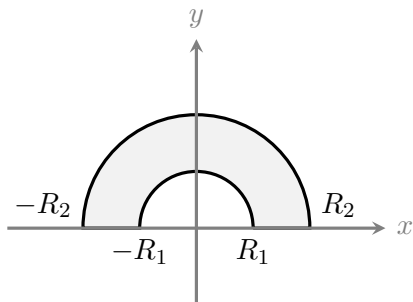
$$0 \leq \theta \leq 2\pi$$

$$S = \int_{R_1}^{R_2} \int_0^{2\pi} r dr d\theta$$

$$S = \left( \int_{R_1}^{R_2} r dr \right) \left( \int_0^{2\pi} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_{R_1}^{R_2} \right) \left( [\theta]_0^{2\pi} \right)$$

$$S = \frac{1}{2} (R_2^2 - R_1^2) \times 2\pi = \pi (R_2^2 - R_1^2)$$



$$dS = r dr d\theta$$

$$R_1 \leq r \leq R_2$$

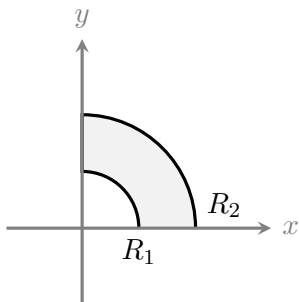
$$0 \leq \theta \leq \pi$$

$$S = \int_{R_1}^{R_2} \int_0^{\pi} r dr d\theta$$

$$S = \left( \int_{R_1}^{R_2} r dr \right) \left( \int_0^{\pi} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_{R_1}^{R_2} \right) ([\theta]_0^{\pi})$$

$$S = \frac{1}{2} (R_2^2 - R_1^2) \times \pi = \frac{1}{2} \pi (R_2^2 - R_1^2)$$



$$dS = r dr d\theta$$

$$R_1 \leq r \leq R_2$$

$$0 \leq \theta \leq \pi/2$$

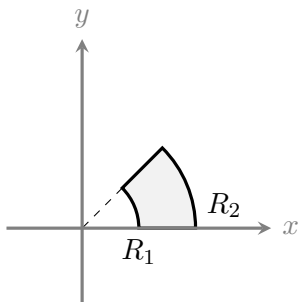
$$S = \int_{R_1}^{R_2} \int_0^{\pi/2} r dr d\theta$$

$$S = \left( \int_{R_1}^{R_2} r dr \right) \left( \int_0^{\pi/2} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_{R_1}^{R_2} \right) \left( [\theta]_0^{\pi/2} \right)$$

$$S = \frac{1}{2} (R_2^2 - R_1^2) \times \frac{\pi}{2} = \frac{1}{4} \pi (R_2^2 - R_1^2)$$





$$dS = r dr d\theta$$

$$R_1 \leq r \leq R_2$$

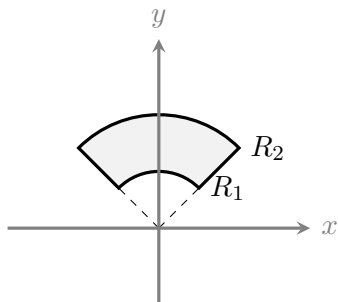
$$0 \leq \theta \leq \pi/4$$

$$S = \int_{R_1}^{R_2} \int_0^{\pi/4} r dr d\theta$$

$$S = \left( \int_{R_1}^{R_2} r dr \right) \left( \int_0^{\pi/4} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_{R_1}^{R_2} \right) \left( [\theta]_0^{\pi/4} \right)$$

$$S = \frac{1}{2} (R_2^2 - R_1^2) \times \frac{\pi}{4} = \frac{1}{8} \pi (R_2^2 - R_1^2)$$



$$dS = r dr d\theta$$

$$R_1 \leq r \leq R_2$$

$$\pi/4 \leq \theta \leq 3\pi/4$$

$$S = \int_{R_1}^{R_2} \int_{\pi/4}^{3\pi/4} r dr d\theta$$

$$S = \left( \int_{R_1}^{R_2} r dr \right) \left( \int_{\pi/4}^{3\pi/4} d\theta \right)$$

$$S = \left( \left[ \frac{1}{2} r^2 \right]_{R_1}^{R_2} \right) \left( [\theta]_{\pi/4}^{3\pi/4} \right)$$

$$S = \frac{1}{2} (R_2^2 - R_1^2) \times \frac{\pi}{2} = \frac{1}{4} \pi (R_2^2 - R_1^2)$$